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Parameter Estimation Based Angular Velocity Observer Design for Rigid-Body Spacecraft

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Abstract— **This paper proposes a simple and physically intuitive approach to the angular velocity estimation problem of rigid-body spacecraft. By taking the advantage of the underlying nature of attitude dynamics, the original state observation problem is equivalently transformed into an easily solvable parameter estimation problem, and global exponential convergence of the observation errors is obtained without using high-gain injection. The main design is extremely concise in overall mathematical formulations and allows great flexibility for implementation, its effectiveness and performance improvements under measurement noises are verified through numerical simulations.**

I. INTRODUCTION

The problem of estimating angular velocity of rigid-body spacecraft has received lots of attentions during the past several decades due to its critical application in many realistic scenarios. Angular velocity is mostly obtained from IMU (inertial measurement unit), while comparing with other commonly used attitude sensors, gyroscopes therein generally have considerably shorter lifespan and are prone to failure [1], and the actual angular rate may also exceed IMU's measurement range during undesired maneuvers. Therefore, accurate estimates of angular velocity can be integrated into alternative or back-up solutions for guidance and control systems, and it can also be used as redundant information for bias calibration and sensor fusion. Moreover, the technics involved also pave the way for designing cost-effective, "gyro-less" devices such as small satellites.

Despite the relative simple formulation of rigid-body attitude dynamics, providing a perfect solution to the aforementioned problem is never an easy task. In the two prevailing classes of methods for angular velocity reconstruction, the numerical differentiation based "derivative approach" [2] typically leads to significantly amplified negative effects of measurement noises and deteriorated estimation accuracy, while angular velocity observers are usually more difficult to be designed. The major obstacle is that the Coriolis term in attitude dynamics is nonlinear in angular velocity, thus making it problematic to dominate resulting error terms globally with static feedback. As a result, the relative fragile asymptotic convergence is usually witnessed when pursuing global stability results [3], [4], [5], where the verification of robustness to uncertainties and separation property for output feedback usually becomes non-trivial. On the other hand, globally exponentially stable (GES) observers do not suffer from these drawbacks, while its design procedure is much more technically involved (interested readers could also refer to [6] for more detailed mathematical explanations). To the best of our knowledge, most of the existing GES results [7], [8], [9], [10] are based on the Immersion & Invariance (I&I) method proposed in [11], but unfortunately, all these methods have to rely on dynamic scaling [12] to overcome the inherent realization issue for higher-order systems in the I&I framework through high-gain injection, and it is well-known that employing high-gain in feedback is particularly undesired when measurement noises are taken into consideration. Several recent interesting advancements on contraction analysis based observer could obviate some of the above problems, while high-gain filtering is still inevitable in these methods to obtain GES [13], [14]. The only exception seems to be the result in [6], where a Kazantzis-Kravaris-Luenberger observer (KKLO) [15] inspired low-gain GES observer is established through using special coordinate transformation and kinematic properties of rigid rotation. However, the design in [6] suffers from limited "bandwidth" when the demand on convergence speed is stringent.

In view of the aforementioned defects in existing designs, we revisited the angular velocity estimation problem in this paper from a completely new perspective. Part of our motivations come from recent efforts on designing parameter estimation based observer (PEBO) for general nonlinear systems [16], where parameter estimation approaches are enabled to solve the state observation problem via the establishment of certain coordinate transformation through solving the associated partial differential equation (PDE). Like KKLO designs, PEBO is born to be high-gain free and has already been applied in several representative cases with promising advantages [17], [18], [19], however, the PDE solvability and existence of the parameter estimator are still the main difficulties for its further generalization to other nonlinear systems. Nevertheless, by taking advantages of the underlying physical nature of attitude dynamics, we are able to find natural coordinate transformation which avoids solving the PDE in original PEBO design, and the resulting parameter estimation problem ultimately becomes a linear regression problem where the regression matrix is always persistently exciting (PE). On this basis, a novel GES angular velocity observer is proposed, where the following major

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contributions and improvements are witnessed:

1) The main design is physically intuitive and free from using any kind of high-gain injection, which is a noteworthy departure from prevailing methods to obtain GES;
2) The proposed observer is developed in an extremely simple and straightforward manner: it is only governed by two key equations, and even all associated analysis is free of using Lyapunov functions;
3) The formulation of the observer is independent of specific attitude representations or parameter estimators, which allows considerable flexibility for extension and implementation.

The remainder of this paper is organized as follows. The model and objective are introduced in Section II. Section III gives the main result of the proposed PEBO design, and simulation results are presented in Section IV to demonstrate the effectiveness of the proposed method. Finally, concluding remarks are summarized in Section V.

II. MODEL AND PROBLEM DESCRIPTION

The attitude dynamics of the rigid-body spacecraft is given by

$$J\dot{\omega} + \omega^{\times} J\omega = \tau, \tag{1}$$

where $J \in \mathbb{R}^{3\times 3}$ is the constant and positive definite inertia matrix. $\omega \in \mathbb{R}^3$ is the unknown angular velocity to be observed, which is defined in the body-fixed frame $\mathcal{F}_B$ with respect to the inertial reference frame $\mathcal{F}_I$, and $\tau \in \mathbb{R}^3$ represents the control input. The skew-symmetric matrix operator $(\cdot)^{\times} : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ is defined such that $x^{\times} y = x \times y$ holds for all $x, y \in \mathbb{R}^3$. Without loss of generality (this will be discussed later), the unit quaternion will be considered as available attitude information, where we denote $q \triangleq \{q_0, q_v\} \in \mathbb{R} \times \mathbb{R}^3$ as the attitude orientation of $\mathcal{F}_B$ with respect to $\mathcal{F}_I$, subjecting to the constraint $q_0^2 + q_v^{\mathrm{T}} q_v = 1$. The kinematic equation of the quaternion representation is given by

$$\dot{q} = T(q)\omega, \quad T(q) = \frac{1}{2}\begin{bmatrix} -q_v^{\mathrm{T}} \\ q_v^{\times} + q_0 I_3 \end{bmatrix}. \tag{2}$$

And correspondingly, we have

$$4T^{\top} T = I_3, \tag{3}$$

where I_3 is the 3-dimensional identity matrix. The rotation matrix R that brings $\mathcal{F}_I$ to $\mathcal{F}_B$ is thus determined as

$$R(q) = I_3 - 2q_0 q_v^{\times} + 2q_v^{\times} q_v^{\times}. \tag{4}$$

The main objective of this paper is to find a specific smooth mapping $\hat{\omega} \in \mathbb{R}^3$ that will guarantee global exponential convergence of the observation error $\hat{\omega} - \omega$ (namely $\lim_{t\to\infty} e^{at}(\hat{\omega} - \omega) = 0$ for some $a \in \mathbb{R}^+$ and for all possible initial conditions) given the accurate information of J and τ. Such problem will be studied under the realistic and commonly used assumption that τ is such that the unknown state ω is bounded by *unknown* constants, and we define such control input belongs to class $\mathcal{U}$ (denoted by $\tau \in \mathcal{U}$). The main focus will be given to the development of new strategies that will lead to simple and physically intuitive GES observer design which avoids using high-gain injection.

III. MAIN RESULTS

A. Motivations

The main motivation of the angular velocity observer design in this paper comes from an interesting observation: the direct estimation problem of ω could be solved in an alternative way if we shift our focus to the estimation of an indirect variable $\psi(\omega, y)$ that contains the necessary information of ω, where y denotes available outputs. And to further recover ω from the estimate of $\psi(\omega, y)$, we need to make sure that ψ is appropriately selected to guarantee the existence of a related mapping ψ^L such that

$$\psi^L(\psi(\omega, y), y) = \omega. \tag{5}$$

In such case, the direct estimation problem of ω can be equivalently transformed into the estimation problem of $\psi(\omega, y)$, which may significantly ease the difficulties in observer design. This idea first appeared in the cornerstone work of Luenberger observer for linear systems [20]. It should be noted that the key of Luenberger's original observer formulation is certainly the above coordinate transformation rather than the well-known pole placement technic, because when considering their extensions to general nonlinear systems, the latter gives rise to the classic high-gain observer designs which rely on increasing observation gain to force the convergence of observation error, while the former ultimately inspires KKLO designs that are aiming at obtaining stable linear error dynamics in new coordinates and thus completely obviates the need of high-gain domination [15].

Inspired by KKLO designs, we have proposed the first "low-gain" GES result on quaternion based angular velocity observer design in [6], where high-gain injection is successfully avoided by estimating a $T\omega$ related term rather than directly estimating ω. However, as mentioned in the introduction, the method in [6] has only limited "bandwidth", i.e., the convergence speed of observation error cannot be arbitrarily increased. We seek to find a new solution to this problem in this paper, and in the meantime, we wish to inherit its advantages on avoiding high-gain injection.

To obtain the desired properties, the indirect estimation of ω will still be taking into consideration, and this time we will estimate the angular momentum of the rigid-body spacecraft in inertial frame $\mathcal{F}_I$. Such consideration is mainly based on the following facts:

1) The difficulties in dominating the problematic Coriolis term in $\mathcal{F}_B$ can be avoided if we shift our interests in the inertial frame $\mathcal{F}_I$, and according to the conservation law of angular momentum, we have the simple enough relationship that

$$\dot{h}^I = R^{\top}\tau, \tag{6}$$

where $h^I = R^{\top} J\omega$ is the angular momentum represented in $\mathcal{F}_I$. It is clear that the nonlinear-in-ω Coriolis

term vanishes in the new estimation problem of h^I, and through using the relationship $\omega = (R^\top J)^{-1} h^I = J^{-1} R h^I$, we could generate estimates of ω with $\hat{\omega} = J^{-1} R \hat{h}^I$, where $\hat{h}^I$ is any accurate estimate of h^I;
2) The original observation problem will actually become a parameter estimation alike problem if we consider a "copy" of Eq. (6), namely

$$\dot{\chi} = R^\top \tau. \tag{7}$$

As a result, the mismatch between χ and h^I becomes a constant parameter, and if we could formulate an appropriate regression equation for this constant, then the original observation problem can be solved with commonly used parameter estimation approaches.

Based on these considerations, we will facilitate the parameter estimation based observer design in the following subsection.

B. Observer formulation

The main PEBO design is now given in the following theorem.

Theorem 1. *Consider the rigid-body spacecraft in Eqs. (1) and (2) verifying the assumption that $\tau \in \mathcal{U}$. The parameter estimation based angular velocity observer is given by*

$$\hat{\omega} = J^{-1} R(\chi + \hat{\theta}), \tag{8}$$

$$\dot{\chi} = R^\top \tau, \tag{9}$$

where $\hat{\theta} \in \mathbb{R}^3$ is any realizable parameter estimation law for the unknown constant $\theta \in \mathbb{R}^3$ that is defined by the linear regression equation

$$\varphi\theta = \dot{q} - \varphi\chi, \quad \varphi = T J^{-1} R. \tag{10}$$

As a result, the proposed observer in Eqs. (8) to (9) guarantees global exponential convergence of $\hat{\omega} - \omega$ for all possible initial conditions if the parameter estimation error $\hat{\theta} - \theta$ does so.

Remark 1. Before proceeding with the proof, we would like to clear some possible misunderstandings.

1) The first misunderstanding may come from the description of "realizable parameter estimation law", as the right-hand side of regression equation in Eq. (10) contains unknown signal $\dot{q}$. Nevertheless, such problem has already been recognized as a well-solved one in the adaptive control/parameter identification literature, where the most popular technic is to use stable linear low-pass filters to generate a new "realizable" regression equation from Eq. (10) such that all signals involved are directly available. We will also briefly discuss on realizations in the next subsection;
2) One may also have suspicions on the existence of a parameter estimator which guarantees global exponential convergence of $\hat{\theta} - \theta$. However, Eq. (3) and $\varphi = T J^{-1} R$ imply that $4\varphi^\top \varphi = R^\top (J^{-1})^2 R$, thus with the positive-definiteness of J and the orthogonality of R, we know that the regression matrix φ is always PE. It then becomes quite easy to design a realizable parameter estimator $\hat{\theta}$ to ensure global exponential convergence of the estimation error: classic methods like gradient descent and least-squares will work as expected with the aforementioned "realizable" regression equation, and many other interesting solutions could also accomplish such task [11], [21], [22];
3) The final attention will be given to the assumption $\tau \in \mathcal{U}$, which is slightly stronger than the forward completeness assumption in many observer designs for general nonlinear systems [23]. However, such assumption does not seem to be overly strong for realistic applications because we do not need to use exact bounds of the unknown state in the main design at all. Furthermore, it is demonstrated in [6] that this assumption may not affect the establishment of the certainty-equivalence/separation property alike results in output feedback.

Proof. Consider the estimation problem of $h^I = R^\top J\omega$, from the previous discussion from Eq. (6) to (7) as well as the formulation of Eq. (9), we know that

$$h^I = \chi + p, \tag{11}$$

where $p \in \mathbb{R}^3$ is the unknown constant parameter to be estimated. Noticing that the kinematics in Eq. (2) yields

$$\begin{aligned} \dot{q} &= T\omega \\ &= T J^{-1} R(R^\top J\omega) \\ &= \varphi h^I = \varphi(\chi + p). \end{aligned} \tag{12}$$

It is clear from Eq. (12) that by defining $p = \theta$, we will arrive at the regression equation in Eq. (10) with

$$h^I = \chi + \theta. \tag{13}$$

The observation error $\hat{\omega} - \omega$ thus becomes

$$\hat{\omega} - \omega = J^{-1} R(\hat{\theta} - \theta). \tag{14}$$

Therefore, if the parameter estimation error $\hat{\theta} - \theta$ converges globally exponentially to zero for all possible initial conditions, the claim of the theorem will then follow from the boundedness of J^{-1} and R. □

C. Discussions

1) Choices on attitude representations: One of the beneficial features about the main result in this paper is the modular alike behavior in terms of the specific attitude representations used in observer design, as different kinematic equations will only make differences on the resulting regression equation, while the rest of the structure of the observer will remain unchanged. For example, if Rodrigues parameter $g = q_v / q_0$ is used to represent the attitude orientation of $\mathcal{F}_B$ with respect to $\mathcal{F}_I$, then the corresponding kinematic equation in Eq. (2) will be replaced by

$$\dot{g} = B(g)\omega, \quad B(g) = \frac{1}{2}(I_3 + g^\times + g g^\top). \tag{15}$$

If we set aside its singularity problem on $180°$ rotation, then it is not difficult to verify that the new regression matrix B is PE, thus the resulting new parameter estimation problem could therefore be easily solved, and similar conclusions can be extended to the usage of Euler angle, modified Rodrigues parameter, and rotation matrix. In summary, it does not matter which attitude representation will be used as long as the selected one guarantees solvability of the resulting parameter estimation problem. Furthermore, in view of the reformulation of kinematics equations in [6], it also seems promising to employ direct vector measurements as available outputs (and thus completely avoids attitude determination) of the proposed observer.

2) Realizations of the parameter estimator: How to use Eq. (10) for parameter estimation will be briefly discussed here. As we have already mentioned, a well-known method to generate "realizable" regression equation is to construct stable linear low-pass filters for the regression matrix and right-hand side of Eq. (10), namely

$$\dot{\varphi}_f = -\alpha\varphi_f + \beta\varphi, \tag{16}$$

$$\dot{z}_f = -\alpha z_f + \beta(\dot{q} - \varphi\chi), \tag{17}$$

where $\alpha, \beta \in \mathbb{R}^+$ are any positive constants. It should be noted that z_f is obtainable because we could rewrite Eq. (17) as

$$z_f(t) = z_f(0) + \beta\big[q(t) - q(0)\big] + \int_0^t -\alpha z_f - \beta\varphi\chi \; d\tau. \tag{18}$$

Therefore, with these definitions, the new "realizable" regression equation can be obtained as

$$\varphi_f\theta = z_f + \epsilon, \tag{19}$$

where $\epsilon = \exp^{-\alpha t}\big[z_f(0) - \varphi_f(0)\theta\big]$ is an exponentially decaying term. Consequently, if the parameter estimation error is defined as

$$\tilde{\theta} = \hat{\theta} - \theta, \tag{20}$$

then the classic gradient descent estimator will yield

$$\dot{\hat{\theta}} = \Gamma\varphi_f^\top(z_f - \varphi_f\hat{\theta}), \tag{21}$$

$$\dot{\tilde{\theta}} = -\Gamma\varphi_f^\top\varphi_f\tilde{\theta} - \Gamma\varphi_f^\top\epsilon, \tag{22}$$

where $\Gamma \in \mathbb{R}^{3\times3}$ is any constant positive definite matrix. Since φ is bounded and always of full rank, from classic results in [24], [25] we know that φ_f will also be bounded and full rank at almost every time instant given the stable linear low-pass filter defined in Eq. (16). Consequently, the global exponential convergence of parameter estimation errors directly follows from such special PE property of φ_f as well as the exponentially decaying property of ϵ, and the overall convergence speed can be made arbitrarily fast by increasing γ. To further improve the consistency of the convergence speed of each parameter estimation errors, the estimation gain in Eq. (21) can be modified as

$$\Gamma = \gamma(\varphi_f^\top\varphi_f + \delta I_3)^{-1}, \tag{23}$$

where $\gamma \in \mathbb{R}^+$ and $\delta \in \mathbb{R}^+$ are any positive constants. In general, δ should be kept to be relative small in order to achieve approximation of Moore-Penrose alike inverse of $\varphi_f^\top\varphi_f$ with $(\delta I_3 + \varphi_f^\top\varphi_f)^{-1}$, while a larger δ could provide more robust results under measurement noises when the amplitude of $\varphi_f^\top\varphi_f$ is relative small.

3) Application in output feedback PD control: The proposed observer is obtained under the assumption $\tau \in \mathcal{U}$, thus to establish certainty-equivalence/separation property alike results with commonly used PD controller like in [5], [6], [7], [8], [9], we may rely on the finite escape time analysis of the closed-loop system to avoid circular reasoning.

Theorem 2. *The output feedback controller*

$$\tau = -k_p q_v - k_d\hat{\omega} \tag{24}$$

guarantees the boundedness of all closed-loop signals with $\lim_{t\to\infty} q_v = 0$ *and* $\lim_{t\to\infty}\omega = 0$ *for any positive constants* $k_p \in \mathbb{R}^+$ *and* $k_d \in \mathbb{R}^+$.

Proof. Suppose that there exist a finite escape time for the closed-loop system at $t = t^* > 0$, then ω, χ, and $\hat{\theta}$ will remain bounded on $t \in [0, t^*)$, and at least one of them will escape to infinity at that time. If this is true for $\hat{\theta}$, then

$$\sup_{t\in[0,t^*)} \tilde{\theta} = \sup_{t\in[0,t^*)} \hat{\theta} = \infty. \tag{25}$$

However, this contradicts with the stability result of $\tilde{\theta}$ in Eq. (22), thus $\hat{\theta}$ and $\tilde{\theta}$ will not escape to infinity at $t = t^*$. Next, the dynamics of χ can be written as

$$\begin{aligned}\dot{\chi} &= R^\top(-k_p q_v - k_d\hat{\omega}) \\ &= -k_d R^\top JR\chi - k_p R^\top q_v - k_d R^\top JR\hat{\theta}.\end{aligned} \tag{26}$$

Since $R^\top JR$ is positive-definite, it is obvious that χ will also not escape to infinity at $t = t^*$ as $\hat{\theta}$ does not. Finally, the relationship

$$\omega = J^{-1}Rh^I = J^{-1}R(\chi + \theta) \tag{27}$$

consists of the final contradiction given the conclusion on χ. Therefore, all closed-loop signals will remain bounded at any finite time. Now consider $t^* = \infty$ and following the same analysis procedure, it is clear that all the above conclusions also hold on $t \in [0, \infty)$. The claim of the theorem then trivially follows because $\hat{\omega} - \omega$ is exponentially decaying, and the full-state feedback form of Eq. (24) is (almost) globally asymptotically stabilizing. □

IV. SIMULATIONS

Numerical simulations will be presented in this section to demonstrate the effectiveness of the proposed observer and to verify some arguments in previous discussions. For all subsequent scenarios, we will use the inertia matrix

$$J = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix}$$

and the initial setting

$$q(0) = \mathrm{col}(\sqrt{1 - 3 \times 0.1826^2}, 0.1826, 0.1826, 0.1826),$$
$$\omega(0) = \mathrm{col}(0.28, -0.36, 0.15) \text{ rad/s}.$$

The proposed observer consists of Eqs. (9), (16), (18), (21), and (23), and the gains are set to be

$$\alpha = 1, \quad \beta = 5, \quad \gamma = 5, \quad k_{q_f} = 2.5, \quad \delta = 0.05. \quad (28)$$

Without loss of generality, the control input is set to be $\tau \equiv \mathrm{col}(0,0,0)$ Nm, and corresponding initial values will be adjusted such that all initial estimates of the angular velocity equals to zero, namely $\hat{\omega}(0) = \mathrm{col}(0,0,0)$ rad/s.

A. Comparisons under measurement noises

One of the main advantages of the proposed PEBO is that GES is established without any kind of high-gain injection, which indicates that improved performance can be expected when measurement noises are taken into consideration. To support this argument, we will compare the proposed PEBO with a representative high-gain injection based GES angular velocity observer design [7]. In order to achieve relative fair comparisons, free parameters in the high-gain injection based observer (we denote it by HGIO) are fine-tuned to obtain similar ideal case performance of proposed observer with the gain settings in Eq. (28). The corrupted measurement of q is given by

$$q_m = \frac{(q+e)}{|q+e|},$$

where the noise $e \in \mathbb{R}^4$ is set to be uniformly distributed among $[-0.025, 0.025]$. Based on these settings, performance comparisons between the two observer designs are given in Figs. 1 and 2.

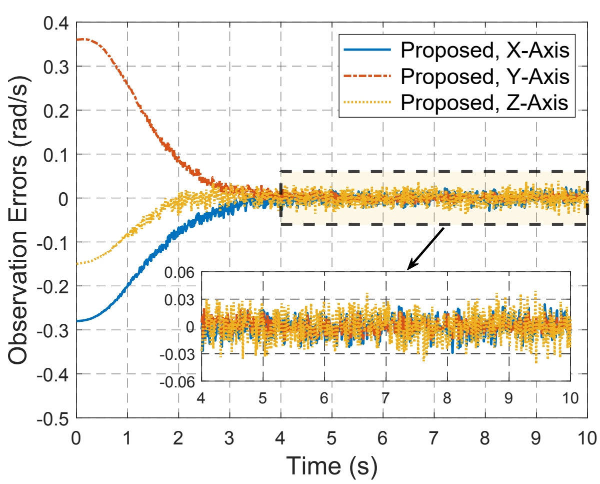

Fig. 1. Performance under measurement noises: proposed observer.

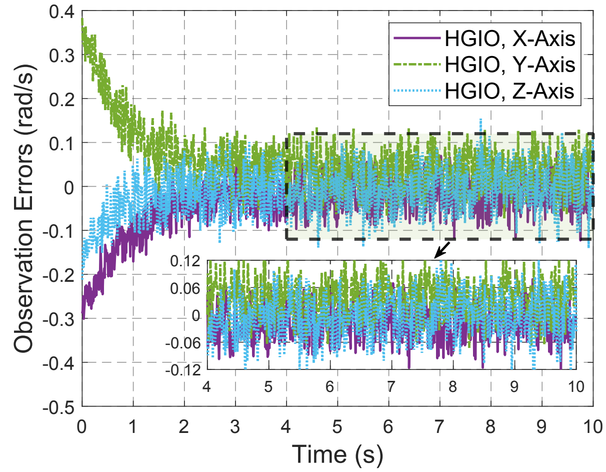

Fig. 2. Performance under measurement noises: reference [7].

As seen from these simulation results, the proposed PEBO behaves significantly better during both the initial transient and steady-state, where the dependency on high-gain injection (Euclidean norms of J and other auxiliary variables are directly used in feedback in [7]) is the main culprit for the accuracy deterioration of HGIO. Instead, the simple, physically intuitive, and high-gain free nature of the proposed observer results in notable performance improvements under measurement noises, and the leading advantage of our method will be further enlarged for systems with higher amplitude of angular momentum.

B. Comparisons on accelerating the convergence speed

Another aforementioned major contribution of this paper is that the convergence speed of the proposed PEBO can be made arbitrarily fast by increasing the observation gain, thus does not suffer from the limited "bandwidth" problem in [6]. To support this argument, performance comparisons between the proposed PEBO and the observer design in [6] with different observation gains (in increasing order) are presented in Figs. 3 and 4.

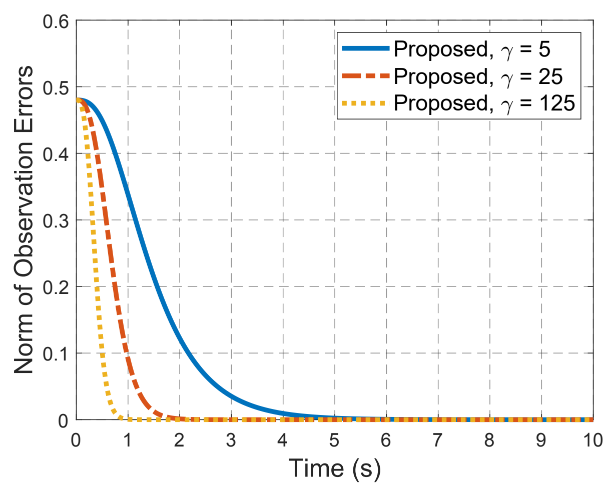

Fig. 3. Performance with different observation gains: proposed observer.

The results in Figs. 3 and 4 clearly verify that increasing the value of γ will lead to significant acceleration on the convergence speed of the proposed PEBO, while similar actions have limited effect on the observer design in [6] when the observation gain reaches a certain threshold.

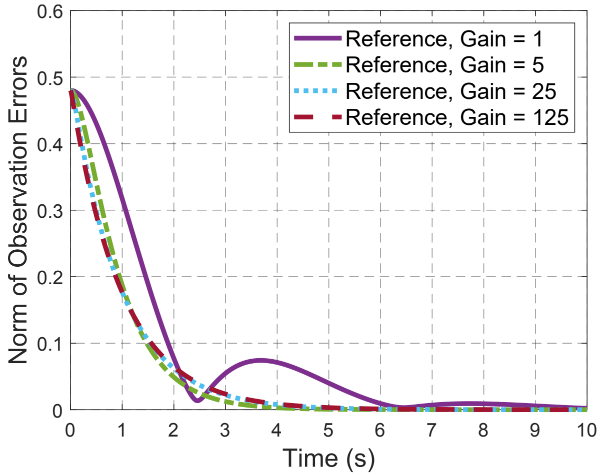

Fig. 4. Performance with different observation gains: reference [6].

V. CONCLUSIONS

A high-gain free, globally exponentially convergent parameter estimation based angular velocity observer is developed in this paper, which provides a radically new solution to the angular velocity estimation problem and also brings useful insights for obviating the obstacles in original PEBO framework. By integrating physical intuition into the observer design, we are able to make the overall mathematical formulation and associated analysis extremely simple and straightforward. Results in this paper established an alternative "low-gain" GES approach on angular velocity observation to our previous contribution in [6], while the limitation on "bandwidth" therein can be easily removed by selecting appropriate parameter estimator. Direct applications in similar state observation problems and generalization of the proposed method to the velocity estimation of Euler-Lagrange mechanical systems are currently pursued.

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