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Optimal Control of Parallel Pressure Filtration Systems

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***Abstract*-Dynamical models exist for the non-complex membrane filtration process which has enabled full scale optimal control development. Parallel pressure filtration systems do have similarities with membrane filtration, but lack models suitable for optimal control and they operate as multiple parallel units and must be treated as a part of a larger plant. A simplified optimal control concept is presented and compared with optimal control of membrane filtration. The optimal control does not use constant filter run cycle times as previous works on membrane filtration, but is based on true feedback control and is capable of finding the optimal run cycle time in varying conditions. Plant-wide control is also addressed in the simulated case example presented in the paper.**

I. INTRODUCTION

Pressure filtration, also known as cake filtration is used in many biochemical production plants to remove impurities from process streams. A filter is a closed pressure vessel containing multiple plates of metal meshes (leafs), on which a layer of solid material, the "cake", consisting of some inorganic material, "clay" is deposited. The liquid to be filtered passes through the cake and the leafs so that the impurities are absorbed into the clay. Clay is continuously fed into the feed stream upstream the filters in a small proportion, and a cake builds up on the leafs thus increasing the differential pressure (DP) over the filter, [2]. When DP reaches a given threshold, the feed to the filter is stopped and the re-initialization (or: recovery, or: off-line) operations commence. First, the filter is emptied from un-filtered slurry, then the filter cake is removed. Then, a pre-coat step is performed, where clean, already once filtered product mixed with clay is pumped through the filter so that a pre-coat cake settles on the cleaned filter leafs, after which next filter run cycle starts.

The complex re-initialization sequences can be automatic or executed as manual operations. Automatic control has since long been recommendable, [9].

Membrane filtration is commonly used in water purification, [5]. No clay is used, and when solid impurities (foulants) have built up on the membrane so that throughput flow rate has decreased down to a given threshold, or when an experience-based time has elapsed, the flow is reversed for a short time period, known as the "backwash" procedure.

Pressure filtration (PF) is often arranged as parallel pressure filtration systems (PPFS) with high throughput capacity.

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Optimal control of such systems seems to not have been deeply analysed in the literature. In [2], the average throughput flow rate of one single filter over one filtering cycle is suggested as the criterion to maximize.

For membrane filtration (MF), optimal control based on the Pontryagin Maximum Principle (PMP) has been published, using only one binary control signal: filtering (u = 1) and backwash (u = -1), [5], [6], [7]. The available dynamical models and simplicity of MF make this possible. The drawbacks of this solution are that a fixed optimal run time for the MF is applied based on a model, which makes it an open loop control, and a fixed operating period over multiple cycles needs to be defined.

The contributions of this paper are: 1) to demonstrate that there exists an optimal filter run time when the filter flow rate is a decreasing function of time 2) to demonstrate that the filter run time optimization can be implemented by a simple real-time control algorithm 3) to use backward-acting inventory control to improve PPFS throughput when rate-of-change constraints are applied on the individual filter flow rates 4) to formulate the throughput maximization of the PPFS in such a way that a fixed operating time period is not required.

In section II the phenomena and models of PF and MF and the optimal control of the same will be presented, in section III we develop a two-level optimal control for a PPFS, and in Section IV we present a simulated case for optimal control under relevant constraints for a PPFS with 5 filters. Section V concludes with discussion and summary.

In this paper "flow" mean volumetric flow rate. The filter cycle consists of the re-initialization (off-line) cycle of length T_o and run cycle of length T_r.

II. FILTRATION SYSTEM MODELS AND CONTROL

A. *Pressure Filtration*

Pressure filtration is based on cake resistance α:

$$\alpha = \alpha_0(\Delta p)^n \tag{1}$$

where α_0 is a material-specific flow resistance, Δp is the DP over the filter and n is the compressibility index in the value range 0.4…0.7. DP evolves over time as follows:

$$\Delta p(t) = \left[\frac{Q(t)}{A^2}\int_{t0}^{t} Q(\tau)d\tau\, \beta\mu(1-n)\right]^{\frac{1}{1-n}} \tag{2}$$

where $Q(t)$ is the flow through the filter, A is the total area of the filter leafs (and cake), μ is the viscosity of the filter feed and $\beta = \alpha_0 c$, where c is the average concentration of solids, including the clay and solid impurities, [2].

Eq. (2) was derived for constant flow Q, [2], [8] and there is no guarantee of it's validity for varying flow.

The average flow through a single filter over one filter cycle is to be maximized, [4]:

$$\max_{T_r} J = \frac{\int_{T_1}^{T_2} Q(t)dt}{T_r + T_o} \quad (3)$$

Where T_1 is the start time of the cycle, T_2 is the end time and $Q(t)$ is known. If there is no flow through the filter during off-line operations, one may write $T_1 = 0$, $T_2 = T_r$. If $Q(t) = Q_c$ is constant, the average flow is $J = Q_c T_r/(T_r + T_o)$. At constant flow it is optimal to run the filter up to the maximum differential pressure limit for maximal average throughput.

B. Membrane Filtration

The liquid that flows through the membrane contains foulants, i.e. solid particles which accumulate on the membrane. The accumulated mass, m, is a function of time for which the models for forward flow during filtering, [5], [6]:

$$\frac{dm(t)}{dt} = \frac{b}{e + m(t)}, \quad m(0) = m_0 \quad (4)$$

and backwash flow, when assumed constant:

$$\frac{dm(t)}{dt} = -am(t) \quad (5)$$

The parameters are defined as $b = \frac{CA^2\Delta P}{\mu\alpha}, e = \frac{R_0 A}{\alpha}, d = A^2\Delta P/\mu\alpha$ and $a = \omega_B Q_B$, where C is the concentration of foulants in the feed, A is the membrane surface area, ΔP is the pressure difference across the membrane, μ is the viscosity of the permeate, α is the specific resistance of the foulant layer, R_0 is the intrinsic resistance of the membrane, ω_B is the detachment resistance of foulants, and Q_B is the constant backwash flow. The flow during filtering is:

$$Q(t) \triangleq Q(m(t)) = \frac{d}{e + m(t)} \quad (6)$$

Using a single, binary control signal $u(t)$: $u(t) = 1$, when the filter is filtering and $u(t) = -1$ for backwash, the accumulated amount of filtered product over a pre-defined time period T is to be maximized:

$$\max_{u(t)} J = \int_0^T [\frac{1 + u(t)}{2} Q(t) - \frac{1 - u(t)}{2} Q_B]dt \quad (7)$$

The optimal control is solved using PMP and provides a switching policy and a singular arc, characterized by the physically unrealizable value $u(t) < 1$, which is interpreted as the *pulse width ratio* of $u(t)$, switching between the values -1 and 1. The pulse duration is not provided by the optimal solution and needs to be found out by simulation. The singular arc is reached typically at a very short time $T_1 \ll T$, using $u(t) = 1$, and the singular arc is left at time T_2, shortly before the end time: $T\text{-}T_2 \ll T$, also with $u(t) = 1$.

The practical interpretation of the solution is that it is optimal to start backwash after a quite short time of filtering, and the repeat the short filter cycles (comprising of run and backwash) until T_2.

A generalization of the optimal control is presented in [7]. The switching character and short run cycles remain optimal for a large class of functions characterizing accumulation and detachment of foulant, provided that $Q(t)$ in (6) is a decreasing function.

C. Pressure and Membrane Filtration Compared

The run time of PF may vary a lot in practical applications, [2], [3], being from 8 to17 hours. A typical time required to re-initialize one filter is 1 hour.

During the PF run cycle it is preferable, [2], [3], [4], to have as constant flow Q(t) as possible. If the flow cannot be kept constant, at least flow increases should be avoided towards the end of the run cycle, while flow decreases are allowed. Constant flow targets could not be met in laboratory tests on PF in [8] and [11] while flows decreased more than 60% during the run cycle.

PF are typically a part of a larger plant for which material balance must be managed, so flow through the PF cannot be freely controlled. The combined task of PF throughput maximization and plant-wide material balance management is considered difficult, [2], [3].

MF is operated with constant pressure difference throughout the run cycle contrary to PF's which run with increasing pressure. MF seem to operate with a strongly decreasing flow profile, even showing 92% flow decrease during the run cycle, [5].

Typical run cycles for MF are 1 hour, and the off-line cycle, i.e. backwash, is less than one minute, [5], [6], [7]. PF and PPFS have high flow throughput while MF seems to be used in applications with much lower throughput, [2],[3], [4], [5].

III . OPTIMAL FILTER RUN-TIME

We set as a target to develop optimal control for a PPFS. The complexity of PPFS and the fact that suitable models like (4) to (6) for MF are not available, we shall adopt a less rigorous approach. The optimal control strategy works on two levels: run-time optimization for individual parallel filters at the lower level and coordination of constraints and plant-wide throughput flow maximization at the higher level.

Backwash in MF is equivalent to a loss of filtered product, whereas pre-coating in PF normally does not mean a loss, while the pre-coat solution can be led to the filtered product. For illustration and comparison, we shall discuss pre-coating loss for PF also.

A. Introductory case

Consider (3) with $T_1 = 0$, $T_2 = T_r$ and $Q(t)$ given. Assume that no constraints (see Section III.C below) exist. Setting the derivative of J w.r.t. T_r to zero yields the extrema of J:

$$\frac{dJ}{dT_r} = \frac{Q(T_r)(T_r + T_o) - \int_0^{T_r} Q(t)dt}{(T_r + T_o)^2} = 0 \qquad (8)$$

The second derivative of J is:

$$\frac{d^2J}{dT_r^2} = \frac{\frac{dQ}{dt}\big|_{t=T_r^*}}{(T_r^* + T_o)} \qquad (9)$$

so we have maximum of J at $t = T_r^*$ if the derivative of $Q(t)$ at $t = T_r^*$ is negative.

If a pre-coat loss of volume of filtered product, V_p, needs to be considered, (3) is written by noting that whatever V_p is, it is a constant not dependent on T_r:

$$\max_{T_r} J = \frac{\int_0^{T_r} Q(t)dt - V_p}{T_r + T_o} \qquad (10)$$

Clearly, $V_p > 0$ affects the optimal run time.

Example. Linearly decaying flow $Q(t) = Q_0 - Q_1 t$. We have $= \frac{Q_0 T_r - Q_1 T_r^2/2}{T_r + T_o}$, so $\frac{dJ}{dT_r} = \frac{Q_0 T_o - Q_1 T_o T_r - Q_1 T_r^2/2}{(T_r + T_o)^2}$

Solving for $\frac{dJ}{dT_r} = 0$ gives $T_r^* = \sqrt{T_o^2 + \frac{2Q_0 T_o}{Q_1}} - T_o$. With pre-coat loss, $T_r^* = \sqrt{T_o^2 + \frac{2(Q_0 T_o + V_p)}{Q_1}} - T_o$

Fig. 1a. shows Q and J for $Q_0 = 30$ m^3/h, $Q_1 = 2$ m^3/h^2 ,$V_p = 0$ and $T_o = 1$ hour, as functions of run time T_r. At the optimum run time, Q and J intersect, which is also obvious from (8).

Note 1. A simple control strategy can be designed, where only flow $Q(t)$ is measured during the run cycle, and the time t at which $Q(t)$ intersects with the integrated flow divided by T_o+t, is the optimal run time and the filter should go off-line.

Note 2. Because $Q(t)$ is to a great extent determined by external circumstances, like material balance control, it may become irregular and generate multiple extrema to (10). Fig. 1 b. shows the linear flow of the example which decreases to a constant value Q_2 for 1.7 hours. If Q_2= 13 m^3/h, the global optimum of J is at T_r = 6.17 hours, but if Q_2 = 11 m^3/h, the global optimum is at T_r = 1.65 hours.

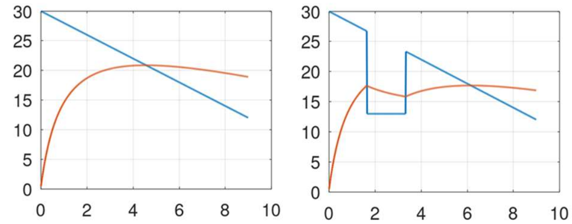

Fig 1 Flow (m^3/h, blue line) and J (m^3/h, red line) as functions of candidate T_r. a: (left) linear flow, b: (right) irregularity in linear flow, Q_2=13 m^3/h.

B. Optimal filter run time

The average flow (3) over one filter cycle is formally not correct to be used when considering multiple filter cycles. In the spirit of optimal feedback control, we must, when optimizing T_r at cycle number "k", consider what has happened in all past cycles 1,2,…,k-1. Rewriting (3), assuming no pre-coat loss, yields:

$$J = \frac{S_c + \int_{T_{1k}}^{T_{2k}} Q_k(t)dt}{T_c + T_{2k} - T_{1k} + T_{ok}} \qquad (11)$$

Where:

$$S_c = \sum_{i=1}^{k-1} \int_{T_{1i}}^{T_{2i}} Q_i(t)dt \qquad (12a)$$

$$T_c = \sum_{i=1}^{k-1} T_{ri} + T_{oi} \triangleq \sum_{i=1}^{k-1} T_{2i} - T_{2,i-1}\,,\ T_{20} = 0 \qquad (12b)$$

Where T_{ri} is the run time, T_{oi} is the off-line time of cycle "i" and $Q_i(t)$,i = 1…k, are known filter flows. Without loss of generality, we may in (11) set $Q_k(t) \to Q(t), T_{1k} \to 0, T_{2k} \to T_r$ and $T_{ok} \to T_o$. T_r is subject to constraints:

$$T_{rN} \le T_r \le T_{rX} \qquad (12c)$$

The unconstrained optimal T_r^* can be solved analogously as in Section III.A:

$$\frac{dJ}{dT_r} = \frac{Q(T_r)(T_c + T_r + T_o) - (S_c + \int_0^{T_r} Q(t)dt)}{(T_c + T_r + T_o)^2} = 0 \qquad (13)$$

In (13), $Q(T_r)T_c - S_c = 0$ if:

1. Cycle number $k = 1$ is considered
2. At cycle $k > 1$, we just disregard the past cycles of the filter
3. If all cycles have been identical, i.e. the flow $Q_i(t)$, T_{ri} and T_{oi} have been identical for all cycles $i = 1,\dots,k$

At the optimal T_r^* we have:

$$Q(T_r^*) = \frac{S_c + \int_0^{T_r^*} Q(t)dt}{T_c + T_r^* + T_o} \triangleq J^* \tag{14}$$

The possibly constrained run time becomes: $T_r^* \to \max(\min(T_r^*, T_{rX}), T_{rN})$.

Note 3: The simple real-time control algorithm in Note 1 above is easily extended to the constrained multiple cycle case. It simply integrates the flow starting from time 0 to present time t, and when $Q(t)$ coincides with the total integrated flow, including S_c, divided by $T_c + T_o + t$, then we have that $t = T_r^*$, and the filter run cycle ends unless minimum time forces to extend run time or T_{rX} forces to stop before the unconstrained T_r^*.

C. *Optimal control of a PPFS*

The run-time optimization of N parallel filters can proceed independently under the condition that min/max constraints on run time for each filter are obeyed. On the higher level, constraints related to usage of process units and equipment related to off-line operations must be taken care of. Typically, [4], off-line operations can be done for only one filter at a time, so simultaneous end of run cycle for two or more filters must be avoided. This can be achieved by starting up the PPFS in a suitable sequence (fig.2) and by calculating T_{rN} and T_{rX} for each filter using a straight-forward look-ahead calculation (not shown here) in case there appears variations in off-line and run times.

The run-time of each filter, T_r, must be at least $(N\text{-}1)T_o$. If $T_r > (N\text{-}1)T_o$, we define $T_a = [T_r - (N\text{-}1)T_o]/N$ which is a periodically occurring time period during which all parallel filters are running, see fig. 2. The total flow through the PPFS shows considerable variations and may indeed challenge overall material balance control.

The higher level PPFS optimal control takes care of calculating the constraints for each individual filter. Using $Q_{kj}(t)$ for the flow of each filter "j" at cycle "k" we have constraints:

$$\Delta p(t) = a_j Q_{kj}(t) \int_0^t Q_{kj}(\tau)d\tau \leq [\Delta p_{Max,j}(t)]^{1-n} \tag{15}$$

$$Q_{Min,k,j}(t) \leq Q_{kj}(t) \leq Q_{Max,k,j}(t) \tag{16}$$

$$D_{Min,k,j}(t) \leq \frac{dQ_{kj}(t)}{dt} \leq D_{Max,k,j}(t) \tag{17}$$

$$T_{rN,j} \leq T_{rj} \leq T_{rX,j} \tag{18}$$

Where (15) is a DP constraint obtained by applying (1) for each filter j: $a_j = \frac{\beta_j \mu_j (1-n)}{A_j^2}$, (16) and (17) are constraints on filter flows and rate-of-change of filter flows. In addition to these, constraints dependent on plant-wide variables may be needed. The criterion to be maximised must be defined, such as the total throughput flow of the system. See Section IV.

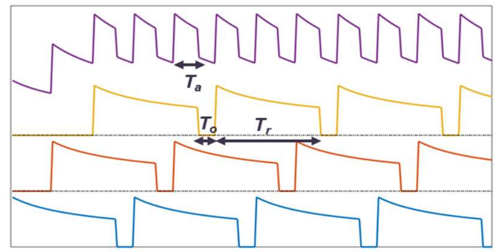

Fig 2. PPFS with three filters starting up, filter flows (blue, red and yellow) as functions of time, and total flow through the PPFS (purple). Principle plot without units.

Note 4: From (15) it can be concluded that when DP approaches the maximum limit, it is favourable that Q_{kj} is small which allows a large value of the integrated flow, thus contributing to increase of average throughput J.

IV. THROUGHPUT OPTIMIZATION OF A PARALLEL PRESSURE FILTRATION SYSTEM

We will discuss a PPFS with $N = 5$ parallel filters as a part of a plant with one filter feed tank, see figure 3.

We add challenge by including fast filter empty during the off-line cycles to the filter feed tank, which causes large temporary increases in the inventory of that tank.

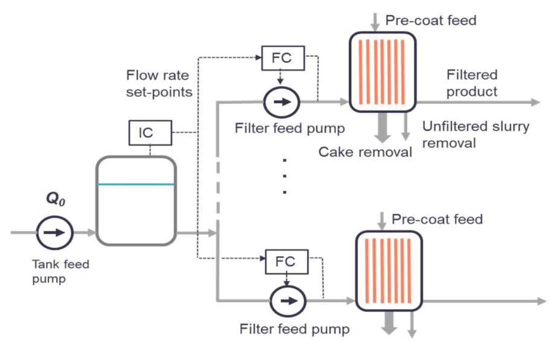

Fig. 3. Process unit with a feed pump, feed tank and a PPFS. IC is an inventory (or level) controller and FC are flow controllers getting setpoints from IC.

A. *Optimal control problem definition*

The problem is to find a maximal *constant* flow Q_0 into the feed tank by optimal choice of filter run-times:

$$Max\ Q_0 \tag{19}$$

Subject to:

$$\frac{dV(t)}{dt} = Q_0 - Q_s(t) + Q_e(t) \tag{20}$$

$$Q_s(t) = K_P\left[(V(t) - V_0) + \frac{1}{T_I}\int_0^t (V(\tau) - V_0)d\tau\right] \tag{21}$$

$$Q_s(t) = \sum_{i=1}^{N} Q_i(t) \quad (22)$$

$$V_{Min} \le V(t) \le V_{Max} \quad (23)$$

and subject to constraints (15) to (18) for each individual filter. Note, that the cycle index "k" can be left away for the higher level plant-wide constraints.

Eq. (20) is the differential equation for the volume balance of the tank. $Q_e(t) = 20$ m^3/h for 15 min. when a filter is emptying at the beginning of the off-line cycle and $Q_e(t) = 0$ otherwise. (21) is the proportional plus integrating (PI) inventory controller of the tank volume $V(t)$, with the flow $Q_s(t)$ as controller output and bound to be exactly the sum of individual filter flows as expressed by (22). $V(t)$ must lie between V_{Min} = 15 m^3 and V_{Max} = 25 m^3. The filters are identical. i.e. have identical values for μ, β, A and n (See (2)). The maximum allowed flow rate for each filter, constraint (16), is $Q_{Max,k,j}(t) = 28 - 2.7\Delta p(t)$ for $\Delta p = 0\ldots\Delta p_{Max} \triangleq 4$ bar, where $\Delta p(t)$ is obtained from (15). $Q_{Min,k,j}(t) = 25$ m^3/h for t = 0…60 seconds and otherwise zero, in order to ensure that filter "j" starts the run cycle with a flow which promotes maximal plant throughput.

The rate-of-change constraint (17) for each filter is:

$$-\delta_1 \le \frac{dQ_{kj}(t)}{dt} \le \delta_2(t) \quad (24)$$

Where δ_1 is a sufficiently large number and $\delta_2(t) = \max\left(0, 0.7 - \frac{1.05}{T_{re}}\right)$: T_{re} is the expected run-time of the current cycle which is equal to the previous actual run cycle of the same filter. The maximum rate of change starts at the value 0.7 m^3/h/min. and decreases to zero at relative time $2T_{re}/3$ (in the run cycle counting from t = 0). Because controller output $Q_s(t)$ is subject to constraints, anti-windup logic must be used, [10]. Details are not presented here due to space limitation. The filters are started with maximal avoidance of risk for simultaneous off-line operations (fig. 2), allowing a minimum T_a = 2 min. The base case for all simulations is chosen as T_r = 8 hours (T_a = 48 min.), because in this time, DP reaches the maximum limit when the filter flow obeys the Δp- dependent maximum flow curve presented above. T_r* is calculated for all filters using the algorithm in Section III.B, Note 3.

All continuous-time models presented above are discretized with a sample interval of 1 minute into a simulation model.

B. Forward-acting inventory control

We use a simple line search to find the maximum constant Q_0. As a reference case, we use no constraints on rate-of-change of filter flows, and with T_r = 8 hours, we obtain the maximal feed F_0 = 85.02 m^3/h. Fig. 4 shows filter 1 and 2 flows, tank inventory and emptying flows from filters 1 and 2. The green arrows illustrate that the emptying takes place as a first step in off-line. Inventory increases rapidly every time a filter is emptying and increases also after that due to less available throughput capability of the PPFS because one filter has just gone off-line.

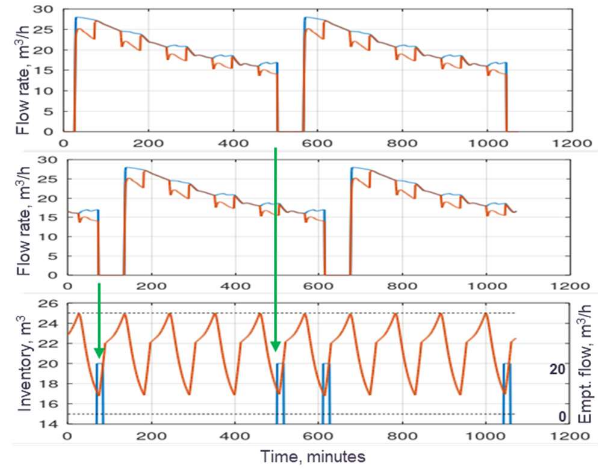

Fig. 4. Top: filter 1 max flow (blue) and actual (red), middle: ditto for filter 2, bottom: tank inventory (red) and emptying flows to tank from filters 1 and 2 (blue; scale is 0 to 20 m^3/h). Limits for tank inventory shown as black dotted lines.

Since there are no rate-of-change constraints, tank IC makes flow increases over the whole filter run cycle. Those sudden increases compensate for filters going off-line.

Now, invoking the filter flow rate-of-change constraints (Section IV.A), the maximum achievable inlet flow becomes Q_0 = 73.76 m^3/h with T_r = 8 hours. The unconstrained T_r* is 217 min., which is constrained to 250 min. by the requirement of T_a = 2 min., whereby the achievable Q_0 becomes 89.97 m^3/h. The remarkable improvement comes from the shorter run time but also from that the time spent in fully constrained flow rate change (zero) for each filter reduces from 160 to 83 min. Fig. 5 shows filter 1 flow for run-times 480 and 250 min. (the flow time series for 250 min. has been shifted for easy comparison). The flows for T_r = 250 min. show only small decreases required by IC because T_a = 2 only, so the time

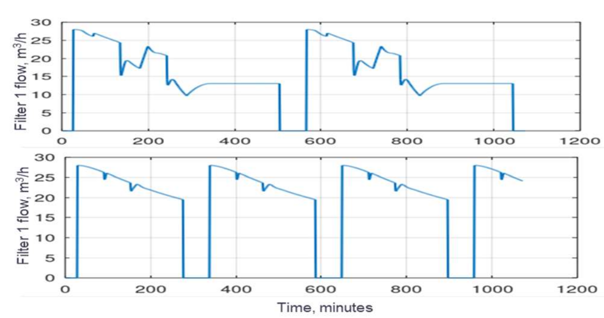

Fig. 5. Filter 1 flow for T_r =480 min. (top) and for T_r =250 min. (bottom).

spent with all filters running, requiring some filters to decrease the flow, is very short.

C. Backward-acting inventory control

Backward-acting control is recommended for inventories upstream from a process bottleneck, [1], which in this case means that IC acts on the tank inlet flow Q_0:

$$Q_0(t) = K_P\left[(V_0 - V(t)) + \frac{1}{T_I}\int_0^t (V_0 - V(\tau))d\tau\right] \qquad (25)$$

Formally, (19) shall also be re-defined to $Max(Q_s)$. The maximization Q_s is now decoupled from material balance control and becomes directly constrained by the DP-dependent maximum flow of individual filters, so the throughput-maximizing strategy is to keep filter flows at their maximum limit over their respective run cycles.

Q_0 = 89.38 m^3/h is achieved for T_r = 480 min. and Q_0 = 93.11 m^3/h for the optimal run-time T_r* = 268 min. which is above the minimum limit 250 min. so the unconstrained run time is the final optimal run time. Table 1 summarizes the results of simulation runs in sections IV.B and IV.C with T_r** = max(T_r*,250). Note, that the maximum DP limit 4 bar was not reached in any of the simulated cases.

Table 1. Maximized average Q_0 for the three cases.

Section, Case	Q_0 (480)	T_r*	T_r**	Q_0 (T_r**)
B, no dQ/dt constraints	85.02	306	306	88.98
B, dQ/dt constrained	73.76	217	250	89.97
C, backward IC	89.38	268	268	93.11

V. CONCLUSIONS

If flows through the filters in a PPFS are decreasing over the run cycle, an optimal run time can be easily found using only the measured flow during the current run cycle and cumulated flows and run / off-line times of past cycles. The presented optimal control of filter run times is truly based on feedback and can adapt to variations in run / off-line times in previous cycles. There is no need to define a fixed operating time period "*T*".

The maximization of throughput of a larger plant of which the PPFS is a part interacts with optimal run time control by constraining the individual filter flows. The paper shows, by the simulated case presented, that backward-acting inventory control can circumvent the specific requirements on filter flows – avoidance of flow increases late in the run cycle- and moreover achieve a superior throughput compared to forward-acting inventory control.

The uncertainties and variabilities of a PPFS, of which unknown composition of the feed flow is one of the most important ones will be reflected on the filter flow and DP behaviour, but one these are measured, optimal control can be implemented. If filter flow rates are very irregular, the cost function may end up having multiple maxima, in which case the optimal run time can be wrongly concluded. Future research should deal with this issue. Another subject for future research is to include operating costs such as energy costs into the optimization problem instead of maximizing throughput at any cost, as we did in this paper.

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