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Comparing Neural Network and Linear Models in Economic MPC: Insights from BOPTEST for Building Temperature Control

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***Abstract*— A data-driven model, in conjunction with economic model predictive control, presents a promising approach to enhance the control of an industrial system with limited development cost. Neural network-based models inherently offer the capacity to identify a wide spectrum of dynamic systems, a pivotal aspect in ensuring a flexible control methodology. However, the training of such neural models requires datasets that are often unattainable in practical scenarios, given that available data is typically confined to the operational data of the system. The literature has shown that linear models are sometimes more relevant in these types of situations, even if they are less flexible. This contribution proposes a comparative study between black-box linear models and neural network-based models. The objective is to evaluate their relevance when used as part of economic predictive controllers in the context of building temperature regulation. The BOPTEST (Building Optimization Performance Tests) benchmark is used for this purpose. Emphasis is placed on different nonlinear model structures to better understand their influence on the results observed in the literature.**

I. Introduction

In a world increasingly marked by the urgent imperatives of climate change and energy transition, the optimization of dynamic system control takes center stage, emerging as a pivotal enabler of resource-efficient and environmentally sustainable technological advancements. Notably, many systems exhibit suboptimal control, often attributed to the limited allocation of resources for deploying state-of-the-art control methods [1]. Among these systems, those with significant global consumption impacts include buildings [1], [2], public swimming pool [3] or waste water treatment plan [4].

The scientific community has introduced new concepts and algorithms to enable greater energy consumption flexibility and enhanced efficiency for equivalent performances [2], [5]. In the HVAC sector (Heating, Ventilation, and Air Conditioning), modern methods can save up to 50% of energy compared to traditional approaches [1]. Unfortunately, these methods are costly to implement due to the addition of sensors, identification experiments, or model design, and hence, they are not widely adopted in practice. This underscores the significance of proposing a versatile framework that necessitates minimal investment to promote wider adoption within the industrial sector.

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With recent advances in non-linear system identification [6], [7] based on neural networks, coupled with the Model Predictive Control (MPC) paradigm [2], [4], it is now feasible to design optimal controls that only require a dataset based on the targeted plant and an estimation of the working cost related to the input signal. This method minimizes the reliance on experts and, ultimately, can facilitate the broader adoption of modern control techniques

Concrete applications of Neural Network (NN) models with MPC have been observed in literature. Afram et al. employed an Economical Model Predictive Control controller (EMPC) with a NN to manage an HVAC system, achieving operating cost savings of up to 70% [2]. A Long Short-Term Memory (LSTM) NN has been applied successfully to identify and control both the four-tank and two tank systems (TS) as well as a Continuous Stirred Tank Reactor [8]. In another contribution, a feed-forward NN was employed to control a four-tank system, surpassing the linear version in terms of steady-state error [9].

While we can celebrate these achievements, certain limitations persist that hinder the widespread success of controls based on NN models. Firstly, highly expressive models demand a substantial amount of well-distributed data to ensure compliance for regulation, given their notable extrapolation limitations [3]. Secondly, and related to the first issue, modern identification approaches predominantly emphasize accuracy metrics to gauge performance. However, recent contributions indicate that while pure accuracy is valuable, it alone is not a sufficient condition for effective closed-loop control performance with EMPC [10], [11].

Black box linear models are less susceptible to extrapolation issues [3], [12] and remain particularly pertinent for control tasks. There are numerous successful applications of MPC using black box linear models in HVAC regulation [4] and chemical process regulation [4], [12]. Such linear models can also yield suitable results without standard identification experiments (such as using a random controller to obtain uncorrelated input from output) in MPC applications [13], [14].

In the long run, unlike highly expressive nonlinear models, linear models do not scale well with larger datasets, potentially limiting the achievement of higher performance [8], [3], [15]. We do not considered, here, techniques that enhance linear expressiveness through adaptive approaches or local linear models [4], [6], as they introduce additional design costs. Furthermore, such linear models may be less accurate compared to grey box models [15].

Nevertheless, there are several contributions that attempt to compare linear and nonlinear black box models in terms of closed-loop performance using predictive controllers. Unfortunately, there is no clear trend that emerges from these studies. Some report better performance with linear models [12], [14], while others highlight the strengths of nonlinear models [8]. This inconsistency can be attributed to the insufficient maturity of nonlinear black box identification methods. Not all comparisons include the latest identification methods, such as [16], and employ equivalent dataset for training.

In response to this situation, this work provides a comprehensive overview of the trends and limitations of data-driven linear and nonlinear methods, emphasizing various nonlinear identification approaches. To carry out this investigation, we employ the BOPTEST control benchmark [5] designed for HVAC systems. This benchmark offers realistic simulations and serves as a platform for implementing EMPC strategies. The selection of building as context is particularly relevant, given the diverse methodologies employed in this field, in contrast to domains such as wastewater treatment plants or swimming pools.

This study presents a comprehensive comparison between linear and nonlinear system identification methods in terms of prediction and closed-loop performance. Integrating recent top-performing nonlinear identification techniques, it offers refined insights over previous research and underscores their real-world applicability. Furthermore, an ablation study of non-linear methods are carried out to furnish concrete recommendations for their application.

The paper is structured to first outline the prevailing models and techniques, followed by an introduction to the methodological approach for comparison. This is followed by an evaluation of the models' predictive and closed-loop performance. Before concluding, an ablation study is presented to discern the impact of the techniques.

II. SELECTED MODEL STRUCTURES AND LEARNING APPROACHES

A. Linear and Non-linear Models: Modern Methods

System identification is a broad field with a wide range of methods [6], [17]. To ensure the clarity of this article, a subset of models was chosen. Furthermore, as we deal with black box models and prediction control methods, only discrete models are considered avoiding so the implementation of numerical integration.

We begin with the Linear State Space (LSS) model employing the subspace identification method [17]. It takes the basic form as follows:

$$\begin{aligned} x[k+1] &= Ax[k] + Bu[k] \\ y[k] &= Cx[k] + e[k] \end{aligned} \tag{1}$$

Here, $y \in \mathbb{R}^{n_y}$, $x \in \mathbb{R}^{n_x}$, and $u \in \mathbb{R}^{n_u}$ represent the observed signals, state signals, and input signals, respectively, while $e[k] \in \mathbb{R}^{n_y}$ represents the observation error. The subspace method allows for obtaining matrix A, B, and C from a given dataset using a specific schema and oblique projection. This model will be taken as the linear baseline for comparison using the MATLAB® system identification toolkit.

Non-linear identification is considered through three nonlinear model structures, each involving neural networks. The Deep Encoder State Space (DESS) method [18] exploits the state space formalism and a nonlinear reconstructibility map to estimate the state:

$$\begin{aligned} x[t] &= \psi_\theta(u[t:t-n_o], y[t:t-n_o]) \\ x[t+1] &= g_\theta(x[t], u[t], e[t]) \\ y[t] &= h_\theta(x[t]) + e[t] \end{aligned} \tag{2}$$

With the three functions ψ_θ, g_θ, and h_θ represent the state transition function, the observation function, and the reconstruction map, respectively. For this model, we will use the associated *Deepsi* [16] open-source library.

To estimate this model, an approximation of the Simulation Error Minimization (SEM) is performed using Multi-Step-ahead Prediction Error Minimization (MS-PEM). This approach has several advantages: it is smoother [19] compared to SEM, more tractable, and equivalent to the (MPC-RI) MPC Relevant Information principle [20]. The MPC Relevant Information principle utilizes the MPC horizon to define the prediction depth used for identification.

The DESS method implementation also introduces a residual block formulation, where the nonlinear function is split into a linear part and a nonlinear part [7]. Some advanced initialization processes have been proposed to improve convergence speed and accuracy [21].

The *Non-Linear Auto-Regressive with eXogenous input* (NLARX) model is a popular model structure that utilizes past delayed input and output signals to predict. It is highly regarded because it can be learned through classic regression tasks and does not require a dedicated observation system. The NLARX model can be considered as a very special case of DESS with a linear reconstructibility map and an imposed choice of the state vector. This model is represented as follows:

$$\begin{aligned} &y[t] = f_\theta(y[t-1:t-n_a], u[t-1:t-n_b]) + e[t] \\ &\text{with} \begin{cases} y[t-1:t-n_a] = [y[t-1]^T \; y[t-2]^T \; \cdots \; y[t-n_a]^T]^T \\ u[t-1:t-n_b] = [u[t-1]^T \; u[t-2]^T \; \cdots \; u[t-n_b]^T]^T \end{cases} \end{aligned} \tag{3}$$

With f_θ representing the fitted NLARX function (with its parameters θ), $y[t-1:t-n_a]$ and $u[t-1:t-n_b]$ are the delayed output and input signals used as regressors for the regression.

This model can be identified using machine learning libraries, but can be enhanced with modern methods like the ResNet architecture (which is equivalent to using discrete derivatives as model output like [22]) and MS-PEM. These two techniques will be implemented subsequently, and will ultimately be involved in the ablation study.

Finally, we introduce another nonlinear model that incorporates multi-step prediction, unlike all previous models that use single-step iterative prediction, as shown in [4]. We will call this model the Multi-Step Neural Network (MS-NN). The concept is to directly predict the entire output horizon using a single feed-forward function. This function is tuned to align with the considered MPC horizon. We employ a neural network and the following formulation:

$$y[t:t+H] = f_\theta(y[t-1:t-n_a], u[t-n_b:t+H-1]) + e[t] \quad (4)$$

Where H is the prediction depth of this model. This last model offers more degrees of freedom since the dynamics of the prediction can vary according to the prediction depth. The last two models are implemented with custom code using the JAX library [23] in Python.

B. Linear vs. Non-linear Models in Closed-Loop Control

Despite numerous contributions [24], [25] that have highlighted the superior accuracy of black box nonlinear models (based on neural networks) in prediction tasks, compared to linear and grey box models, few comparisons have been made in a closed-loop context. In the context of closed-loop comparison, establishing clear dominance of a strategy over another requires further refinement. Many research findings [4], [11], [12], [14], [26] have shown that control performance tends to be inferior when employing non-linear black box models faces to traditional Linear State Space (LSS) or Auto-Regressive models. This phenomenon is elucidated in [10], which shows that achieving superior prediction accuracy within a given test set is not a sufficient guarantee for superior MPC performances. This observation aligns with the extrapolation challenges that neural networks and other highly expressive models can encounter, as discussed in, for instance, [3].

However, certain studies [9], [8] have demonstrated that nonlinear models can lead to enhanced closed-loop performance in certain specific contexts. For instance, Jung et al. [8] employ LSTM for controlling Continuous Stirred-Tank Reactor (CSTR) systems, while Blaud et al. [9] utilize the ResNet architecture and nonlinear predictive control for tank reservoir systems.

The absence of a clear global trend can be attributed to various factors. Firstly, disparities exist in assumptions across studies regarding state measurability. Specifically, while some contributions assume fully measurable states, often resulting in more favorable outcomes with neural network-based models (e.g., [9], [8]), others do not make this assumption. Secondly, there is variability in the experimental design used to feed identification algorithms. Some studies employ random sample experiments, which can enhance the extrapolation performance of nonlinear models, while others opt for more realistic identification experiments. Among the latter notably [14] and [26] have reported unsatisfactory control performance when implementing neural networks as models. Thus, differences in state measurability assumptions and experimental designs collectively contribute to the absence of a trend universally applicable.

This situation is exacerbated when considering the methodological approaches of papers focusing on non-linear methods. Unfortunately, studies spotlighting modern nonlinear methods, such as [8], do not engage in realistic identification experiments, complicating potential conclusions. Conversely, those incorporating realistic identification experiments, such as [4], [12], [14], display non-linear methods that underperform in prediction compared to linear models even before engagement in closed-loop experiments. This performance gap is inconsistent with papers that focus only on prediction performance, such as [24] and [25], where the linear model is beaten most of the time in prediction.

When utilizing nonlinear methods with neural networks, it is essential to conduct hyperparameter optimization (as demonstrated in [9]) and also to present outcomes with multiple trained models to estimate the variance of results stemming from the stochastic nature of training, as noted in [14]. Unfortunately, both of these aspects can be resource-intensive to obtain, which is why they are often lacking in most of the previously mentioned contributions. In the following we try not to avoid these important questions.

III. Methodology

A. Hydronic testcase

The testcase *BESTEST Hydronic Heat Pump* is selected from BOPTEST [5]. It has been studied using various methods, including reinforcement learning [27] and traditional RC grey box models [5]. It is based on the standard BESTEST 900FF structure from the ASHRAE standard [28]. This model represents a simplified version of a residential house for five members. The house consists of a rectangular single zone measuring 12 by 16 meters with a height of 2.7m and an equivalent window on the south side that is 24 m^2.

BOPTEST also offers a realistic controller based on a PID (Proportional-Integral-Derivative) with fixed occupied and unoccupied temperature setpoints. It will be considered as default controller. This type of controller is often considered as a practical way to generate an initial dataset [5], [14], [26], [27], since it maintains the operating requirements. BOPTEST proposes different control criteria to compare different strategies. The total economical cost of the experience and the total discomfort will be used to compare and analyze closed-loop performance. These metrics are directly calculated using BOPTEST and can be easily compared with results from other contributions.

The test scenario includes several inputs and outputs to consider to get optimized predictive models. It also provides various temperature sensors, solar radiation, inhabitant heat perturbations, and more. For the control aspect, we have u_{hp}, u_{pump}, and u_{fan}, which represent the heat pump power, the recycling pump activation, and evaporator fan activation, respectively. To align this study with other works, we adopt the same input/output identification setup as described in [5]. Only the heat pump is controlled, and the last two signals are set to 1 if the heat pump is active (for more details, please refer to [5]).

We also use the same predictive control strategy based on EMPC (Economical Model Predictive Control). The formulation is described in Equation 5. With P_{pump}, P_{fan}, P_{hp} representing the pump, fan and heat pump power respectively. p represents the electricity price according to the simulation profile. f_θ is the black box model to use (one of those introduced in section II-A). The variable w serves as a slack variable, penalizing temperatures lower than the conformance limit T_{conf}.The parameter Q weighs this penalty in the total cost of the MPC formulation.

$$\text{argmin}_{u[1:H]} \sum_{k=1}^{H} p[k](P_{hp}[k] + P_{fan}[k] + P_{pump}[k]) + Qw[k]$$
$$\text{with} \begin{cases} \hat{y}[k+1|t] = f_\theta(\hat{y}[k+t|t], u[k]) \\ w[k] + \hat{y}[k+t|t] \geq T_{\text{conf}}[k+t], \; w[k] \geq 0 \\ \hat{y}[t+0|t] = y[t] \\ P_{hp}[k] = h(u_{hp}[k], y[k]) \\ P_{pump}[k] = 500W \;\text{ if }\; u_{hp}[k] > 0 \;\text{ else }\; 0 \\ P_{fan}[k] = 40W \;\text{ if } u_{hp}[k] > 0 \;\text{ else } 0 \end{cases} \tag{5}$$

The upper limit is ignored because it does not limit the control strategy aimed at minimising heat production (to reduce energy consumption). The Fig. 1 shows the two bounds evolving according to the time. We have selected $Q = 20\$\text{K}^{-1}\text{H}^{-1}$, which corresponds to an equal weighting of the standard regulator performance between its operational cost and constraint violation. To obtain the estimation of the heat pump consumption, a linear regression is performed using the control signal and inside temperature. This regression has a coefficient of determination (R2) of 0.94, which is sufficient for control purposes. According to the results with the grey box [5], we choose a sampling rate of 900s with a time horizon of 12h. We also assume a full future information configuration [29] which means that we consider a perfect weather forecasting to contain experiment complexity.

This EMPC formulation is challenging to solve due to the mixed integer formulation of P_{pump} and P_{fan}. To maintain the lowest possible complexity, as done in [5], the problem is relaxed using a sigmoid function to ensure a smooth transition from $u_{hp} = 0$ to $u_{hp} = 0.1$. The IPOPT continuous nonlinear solver with the Pyomo interface [30] are used to solve the EMPC problem in closed loop.

B. Data sampling and training process

When using black box identification, it is very important to specify which data is used and what experiments are done to generate the training dataset. Contrary to the traditional approach using proper random signals (as in [8], [11]), here we adhere to a protocol that can be deployed in an operating system. As in [27] and the idea proposed in [13], an initial training dataset can be sampled using only the original controller of the plant. This is made possible by multiple setpoints and bounded control which bring a non-zero data distribution of the temperature to regulate. This initial dataset is called *Restricted Dataset*.

To provide more insight into the power of the non-linear identification schema, we also generated a second dataset (as suggested in [13]), using the EMPC formulation (Equation 5) and the linear state space model (Equation 1). This approach remains, due to suitable results with the linear model (see section IV), compliant with the regulation requirements. This second dataset, named *Extended Dataset*, consists of a part with the native controller and a part with the linear MPC-based controller. The aim is to offer two different datasets that can be acquired without any specific identification experiments while remaining compliant with the control requirements.

The benchmark provides only one year of real weather data collected in Belgium, which limits the dataset size. To avoid complexity, data from the entire year will be collected for each dataset to generate global models (valid for all seasons). The *restricted dataset* will have one year of data, and the *extended dataset* will span two years (one for each controller). This is a crucial factor in building modeling, as seasons can significantly affect prediction performance [15]. Assuming a global model can substantially reduce model complexity compared to more intricate approaches, such as having multiple models for each season, which would increase the engineering effort.

The benchmark testcase is composed of two test sections: the *typical heat day* and the *peak heat day*. Both sections represent a periods of 2 weeks that are used for the closed-loop simulation. It is important to exclude these sections from the training dataset, as is commonly done.

Hyper-parameter (HP) optimization is conducted for each model. A grid search approach is used for the linear models, and a random search approach is applied with around 300 HP trials. To avoid any bias in the validation set selection (since our dataset is composed of only one year), a cross-validation scheme is employed. This approach tests a set of HP on 5 different pairs of training and validation datasets. In this study, we have chosen a validation dataset of 6 weeks. The Root Mean Square Error (RMSE) over the MPC horizon is used for model evaluation (see Equation 6). The average of the results for the 5 models is utilized to quantify the performance of the parameter set.

C. Accuracy and control criteria

Each model will be evaluated firstly in multi-step prediction according to the MPC horizon (12h or 48 steps). This evaluation is done with the test section, which comprises the two periods of the given year (*typical heat day* and *peak heat day*). To provide a clear analysis in terms of overfitting, generalization properties etc., we generated test data using three different controllers: The original controller provided by BOPTEST (test section from *restricted dataset*), a LSS-MPC based on Equation 5 (test section from *extended dataset*), and a random controller (that provides random amplitudes with random hold duration). The prediction performance is reduced to a scalar using to the following equation (with H representing the MPC horizon size and K the dataset size):

$$\mathcal{L} = \frac{1}{H} \sum_{i=1}^{H} \sqrt{\sum_{k=0}^{K-1-H} \frac{||\hat{y}[i+k|k] - y[i+k]||_2^2}{K-H}} \tag{6}$$

Next, models are evaluated in closed-loop experiments using different price signals. BOPTEST provides three price senarios: constant price, dynamic price, and highly dynamic price. The first scenario is straightforward, the second is the

Models	Dataset	Nominal controller	LSS MPC controller	Random controller
LSS	Restricted	0.30 ±0.00	0.52 ±0.00	0.41 ±0.00
	Extended	0.62 ±0.00	0.98 ±0.00	0.62 ±0.00
NLARX	Restricted	0.09 ±0.01	0.56 ±0.10	1.05 ±0.20
	Extended	0.20 ±0.04	0.22 ±0.05	0.48 ±0.08
MS-NN	Restricted	0.09 ±0.01	0.38 ±0.01	1.19 ±0.07
	Extended	0.10 ±0.01	0.16 ±0.01	0.99 ±0.05
DESS	Restricted	0.13 ±0.02	0.52 ±0.04	1.73 ±0.04
	Extended	0.29 ±0.06	0.34 ±0.07	1.21 ±0.18

TABLE II: 12h ahead RMSE (Mean. ± Std.) for all models using both restricted and extended datasets results the test set. Extrapolation test sets are shown in orange.

most usual with a price for both day and night periods, and the last is corresponds to the real-time market price. The selected control metrics are the average cost of the regulation and total discomfort for inhabitants. In addition to the two metrics generated by BOPTEST, we introduce a third that merges both objectives using the selected weighting factor Q (from equation 5). This final cost uses the MPCs formulation to compare MPC with their own criteria (also using the heat pump power model). This last metric can be introduced as follows (where Exp_{size} is the experiment size) :

$$\begin{aligned} C = \Delta T \sum_{k}^{\text{Exp}_{\text{size}}} p[k](P_{hp}[k] + P_{fan}[k] + P_{pump}[k]) \\ + Q(T_{conf}[k] - T[k])\mathbb{1}_{T_{conf}[k]>T[k]} \end{aligned} \quad (7)$$

As introduced in [11], we also propose the *n-Step ahead Planning Deviation*. This metric measures the discrepancy between the trajectory planned by the MPC controller and the trajectory realized in closed-loop. The deviation for temperature can be computed as follows:

$$\mathcal{L}^y(t) = \sqrt{\sum_{k=0}^{\text{Exp}_{\text{size}}-1-t} \frac{||\hat{y^*}[t+k|k] - y[t+k]||_2^2}{\text{Exp}_{\text{size}} - t}} \quad (8)$$

With $\hat{y^*}[t+k|k]$ and $y[t+k]$ representing the optimal planning of the open-loop optimization for the time $t+k$ based on observations at instant k, and the actual outcome at instant $t+k$, respectively. In fact, $\mathcal{L}^y(t)$ represents the discrepancy w.r.t the prediction depth (t). This criterion is crucial for determining whether the model mismatch within the optimization process is significant. Here, a zero *n-Step ahead Planning Deviation* is impossible due to the bounded horizon without any terminal cost.

IV. Model Performance

The HPs are optimized using the restricted dataset, and they are listed in Table I. The training horizon specifies the number of prediction steps used for model fitting. The L_1 weight represents regularization using the absolute function of the NN weights. All LSS parameters are linked to the MATLAB API for the *ssest* function.

A. Prediction accuracy

The initial step is to compare model accuracy using both the restricted and extended datasets according to different testsets. TABLE II regroups all these results, with a prediction horizon of 12h. Testsets based on controllers that are not present in the training dataset are highlighted in orange.

LSS parameters :
- n_o: 2, n_x: 2
- focus: *simulation*
- enforce stability: *True*
- $n_4$weight: *MOESP*

MS-NN parameters:
- #layers: 1
- #neurons: 115
- n_o: 6
- Act. func.: *elu*
- Init. weight: *he_uniform*
- L_1 weight: 4.13×10^{-6}
- Learning rate: 9.85×10^{-4}
- Batch size: 64
- #epochs: 1174

NLARX parameters :
- #Layer: 3, #neurons: 9
- n_a: 4, n_b: 5
- Act. func.: *tanh*
- Init.weight: *glorot_uni*
- L_1 weight: 8.30×10^{-5}
- Training horizon: 23
- Learning rate: 0.0023
- Batch size: 512
- #Epochs: 1166

DESS parameters:
- #Layer: 3,
- #neurons: 64
- n_o: 5, n_x: 5
- Training horizon: 15
- Act. func.: *elu*
- Learning rate: 3.53×10^{-5}
- Batch size: 256
- #Epochs: 373

TABLE I: HPs used in the numerical comparison.

In the restricted dataset, the linear model displays poor interpolation accuracy (with data generated using the nominal controller) compared to all non-linear models. Nonetheless, its accuracy remains relatively consistent in the extrapolation regime with data generated by both MPC and random controllers. This confirms that the linear model's ability to maintain consistent extrapolation. Conversely, non-linear models see an increase ranging from 4 times to 7 times when transitioning from data generated by the nominal controller (which involves interpolation) to the MPC controller (which involves extrapolation). This effect is even more pronounced with the random controller, leading to a broader temperature distribution. It is noteworthy that DESS models attain accuracy comparable to the linear model when using the MPC-based test set.

In the expanded regime, the linear model struggles, not being adept at managing a dataset that is twice the size of the restricted dataset. This extended dataset proves advantageous for all non-linear models, which surpass the linear ones in both original test sets. MS-NN delivers the finest interpolation accuracy in comparison to NLARX and DESS. This is attributable to its non-iterative nature which avoids the accumulation of errors.

In the last test set using a random controller, the NLARX model demonstrates the best non-linear generalization property by achieving results that closely resemble the performance of the linear model, contrary to DESS and MS-NN. This may suggest a more sample-efficient algorithm with superior generalization. This observation aligns with the model's high level of constraint in terms of expressivity.

Based solely on these prediction accuracy results, it is difficult to predict any closed-loop results. The linear model does not offer competitive performance but have good generalization capability as its accuracy through different test-set is stable.

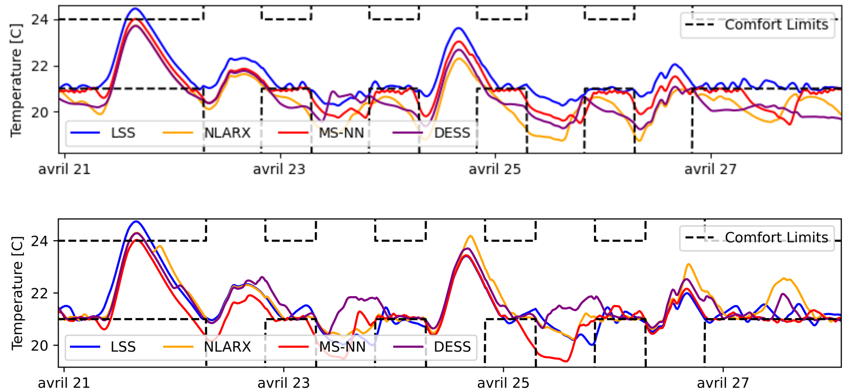

Fig. 1: Experimental results for the *typical heat day* scenario using a dynamic price signal. The upper graph represents results with the restricted dataset, and the lower one represents those using the extended dataset.

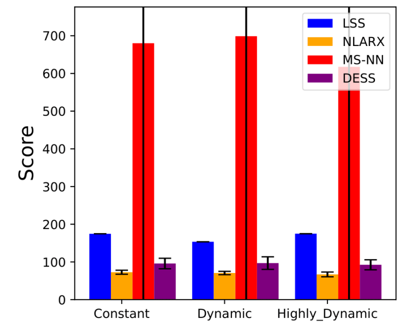

Fig. 2: *Typical heat day* scores (see Equation 7) for selected black box models (boxes stand for ± Std.).

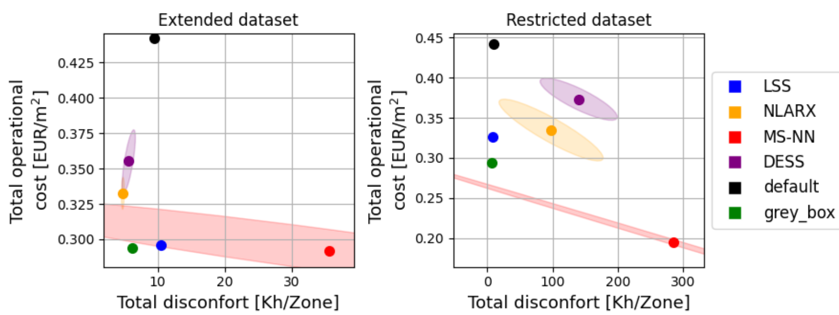

Fig. 3: BOPTEST KPI obtained with selected models using the extended and restricted dataset. Variance estimated using the five different trainings. Score computed with the scenario *typical heat day* and *dynamic* price signal.

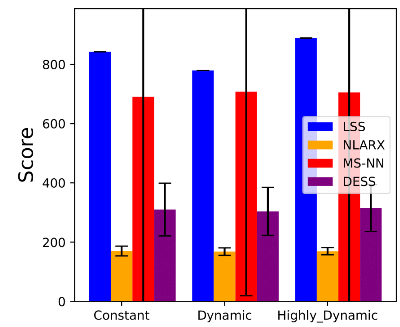

Fig. 4: *Peak heat day* scores (see Equation 7) for selected black box models (boxes stand for ± Std.).

B. Closed loop performance

Closed-loop experiments are conducted for each BOPTEST scenario and price signal five times per type of model (once per trained model).

Fig. 1 shows time results from two experiments using either the restricted dataset or the extended one. From a qualitative point of view, the experiment with the restricted training dataset favors the linear model. All non-linear models produce static errors during the occupancy period (when the comfort lower limit is high) and poor anticipation during constraint changes. On the other hand, linear models offer relevant performance with a slight temperature decrease when possible and an almost stable temperature during the day period. In the second experiment, we observe a substantial improvement for DESS and NLARX models, which stabilize temperature perfectly. NLARX also succeeds in anticipating lower bound fluctuations and manipulating the price signal by shifting load consumption.

If we take a look at the BOPTEST KPIs (Fig. 3), this observation is confirmed. The LSS model with a restricted dataset stays close to the grey box performance from [5], in contrast to non-linear models, which cause a lot of discomfort. The result variance with a restricted dataset is quite high, which reinforces the unrealistic usage of neural network-based models with limited operating data for control. Nonetheless, with an extended dataset, the trend reverses; the NLARX and DESS models can outperform the linear model and provide competitive results against grey box models. This change in performance, influenced by the quality of the datasets, could potentially explain the results that differ in the literature when comparing linear models to neural network-based models.

As we are engaged in a multi-objective performance comparison task, we project the results using the chosen constraint violation cost. This approach allows us to compare models absolutely. We limit our plots to the extended dataset because evaluating the performance of nonlinear models with the restricted dataset would be useless (linear model is the best by far). Fig. 4 and Fig. 2 show, respectively, all performances with their standard deviations for *typical heat day* and *peak heat day*. NLARX and DESS outperform LSS and MS-NN in all price and scenario configurations. The NLARX results are very low and with little variance, outperforming all other models. We observe that DESS is behind NLARX in terms of variance and average performance.

The results from MS-NN can be disappointing, as it achieved the highest accuracy with the nominal and MPC-based test set but failed to produce satisfactory results in the closed-loop scenario. The planning mismatch can be utilized for better understanding. Fig. 5 shows the planning mismatch against closed-loop performance for all models. This curve, which varies with time, is crucial for understanding closed-loop results. As anticipated, NLARX and

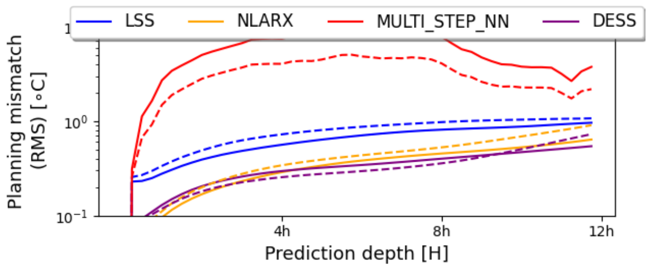

Fig. 5: Model mismatch with respect to the planning prediction depth. Solid line for *typical heat day* and dashed line for *peak heat day*

DESS planning exhibit high short-term accuracy, which can be instrumental in avoiding constraints violation. Moreover, they are the best in both the mid-term and long-term, aiding in the performance analysis in terms of total cost (because good long-term planning allows a transfer of power to exploit the volatility of electricity prices). In contrast, MS-NN planning systematically exhibits inaccuracies across the entire planning range. The inadequacy of planning increases rapidly with the prediction horizon, which may explain the significant discomfort.

From these results, the importance of dataset selection becomes evident. If we consider low-cost and large-scale deployment, the prospect of collecting data and directly using neural models is in the authors' opinion unrealistic. Nevertheless, as suggested in [13], it is conceivable to adopt a protocol in which black-box linear models are initially employed to generate supplementary data while satisfying control specifications. In a subsequent phase, as the operational dataset becomes more diverse, neural models could offer a viable solution to enhance control further.

These results also elucidate why some papers, like [4], employing less optimal neural network architectures for control (such as MS-NN), have found that linear models outperform even when paired with identification experiments. The NLARX model outperforms DESS, which can be attributed to the greater expressivity of DESS and the presence of a nonlinear reconstructibility map that introduces bias in the estimated state.

V. Ablation Study of the NLARX Model

We propose an analysis of the respective influences of the residual approach and the use of MS-PEM on the NLARX model on the observed results.

We follow the same protocol as before to determine the best hyperparameters and train models using both the restricted and extended datasets. Our comparison covers both prediction and control. We have adopted the following naming conventions: 'NLARX' for the standard method, 'NLARX_W_R' for the NLARX model that omits the residual formulation (fitting $y[k+1]$ instead of $y[k+1]-y[k]$), and 'NLARX_W_H' for the NLARX model trained using a one-step-ahead criterion as opposed to MS-PEM.

To commence the presentation of the results, Table III provides a summary of the predictions for the entire MPC horizon (12 hours) using the parameters from the previous dataset. In terms of interpolation performance, NLARX_W_R achieves the lowest errors, which can motivate its use in closed-loop performance. However, in the extrapolation regime (restricted dataset and LSS MPC-based test set), it displays bad results, indicating overfitting to the operating dataset. On the other hand, NLARX_W_H provides similar results in several metrics compared to NLARX. It achieves better extrapolation performance with the restricted dataset and LSS MPC controller based testset but worst with the random controller based testset.

Models	Dataset	Nominal controller	LSS MPC controller	Random controller
NLARX	Restricted	0.09 ±0.03	0.56 ±0.23	1.54 ±0.48
	Extended	0.20 ±0.09	0.22 ±0.10	0.52 ±0.06
NLARX_W_H	Restricted	0.13 ±0.05	0.33 ±0.11	2.71 ±0.41
	Extended	0.15 ±0.06	0.19 ±0.07	0.59 ±0.23
NLARX_W_R	Restricted	0.09 ±0.02	1.07 ±1.04	6.74 ±2.11
	Extended	0.10 ±0.03	0.17 ±0.04	1.57 ±0.88

TABLE III: 12h ahead RMSE (Mean. ± Std.) for different NLARX training configurations using different testsets. Extrapolation testsets are shown in orange.

We observe significant differences in the HP choice when comparing NLARX_W_H and NLARX. For instance, using the MPC-RI criterion, NLARX yields a delay window of $n_a, n_b = 4, 5$. In contrast, NLARX_W_H results in $n_a, n_b = 1, 3$. This discrepancy might explain their similarity in extrapolation (even though NLARX uses a more sophisticated method), since a reduced NN's input size tends to favor extrapolation.

The next step is to compare closed-loop performance. Table IV summarizes results obtained with the *typical heat day* scenario and dynamic price signal. As expected, all models fitted using the restricted dataset yield week results. With the extended dataset, the NLARX model achieves a lower experimental cost, outperforming NLARX_W_R and NLARX_W_H. The limited extrapolation capability of NLARX_W_R is reflected in its results, which remain quite high and are not suitable for practical usage. NLARX proposes an improvement of around 10% in its cost compared to NLARX_W_R.

To conclude this section, using a residual block (or predicting the discrete derivative) in the NLARX architecture is essential to compete against a linear model when considering the operating dataset. Utilizing MPC-RI offers a some improvement in closed-loop experiments but yields mitigated results in extrapolation when considered.

Models	Restricted Dataset	Extended Dataset
NLARX	1974.67 ± 1423.01	$\mathbf{70.56 \pm 4.45}$
NLARX_W_H	$\mathbf{296.76 \pm 177.97}$	76.49 ± 3.85
NLARX_W_R	9063.79 ± 4446.84	308.24 ± 176.86

TABLE IV: Closed-loop score based on the ablation study from Equation 7 (Mean. ± Std.)

VI. Conclusion

This study focused on linear black box models and neural network-based models selected for practical applications and wide deployment in control. To achieve this, it carried out experiments on the BOPTEST benchmark. In light of the models and results explored, dataset selection has been shown to be a critical factor for model accuracy. Classically, linear models have proven to be a viable and satisfactory solution for the type of process considered, especially when dealing with limited data sets. NN-based models have shown improved performance in constrained control when applied to sufficiently diverse operating datasets. The model's architecture and the selection of a suitable training method were crucial for achieving high performance in prediction and closed-loop control. The NLARX architecture with residuals and the use of the MS-PEM criterion demonstrated their superiority over other methods. Its results confirm the possibility of using models based on neural networks in practical scenarios, provided that care is taken to make choices adapted to the type of problem considered, consisting here of improving the control of dynamic industrial processes.

There are opportunities to improve neural network-based models, particularly when starting from limited training datasets. One approach is to incorporate fundamental physics knowledge, as demonstrated in [31]. The main goal is to achieve a high-performance non-linear MPC, proceeding to continuous and secure improvement starting from constrained datasets. Future contributions could extend this comparison by considering diverse setpoints and uncertainty in identified models.

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