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Data-Driven Adaptive Control for unknown underactuated Euler-Lagrange Systems

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Abstract— **In this paper, a data-driven adaptive control approach is developed for unknown underactuated Euler-Lagrange systems. The proposed approach can deal with the nonlinearity and handle unmodelled dynamics, model uncertainties and unknown disturbances in underactuated systems. At first, coupled sliding variables are defined to combine the dynamics of actuated and unactuated states. The time-delayed estimation (TDE) technique is applied to deal with all the unknown factors in the dynamics of sliding variables. A constant gain matrix is the main design parameter and influences both the closed-loop stability and the tracking performance. The data-driven approach developed in this paper can find the constant gain matrix directly from the input and output data without any knowledge of the inertia matrix. To deal with the TDE error, an adaptive sliding mode control is integrated. The proposed approach is illustrated with an example of an offshore boom crane.**

I. INTRODUCTION

Euler-Lagrange systems [1] exist widely in many practical systems in the real world [2], [3]. Underactuation arises in Euler-Lagrange systems whenever the number of independent control inputs is less than the degree of freedom, such as cranes [4], satellites [5], underwater vehicles [6], unmanned aerial vehicles (UAV) [7], [8] and mobile robots [9]. Though underactuated systems have shown the merits in terms of simpler structure, less cost and energy consumption and better operational flexibility, the controller design for underactuated systems is still a challenging task [10], [11].

To guarantee the stability and controllability of nonlinear underactuated Euler-Lagrange systems, different control approaches have been introduced. For instances, plenty of remarkable works including transformation into fully actuated form [12], energy-based control method [13], backstepping control methods [14], model predictive control [15] have been proposed for the underactuated systems. However, the precise model and the accurate system parameters are required. To deal with model uncertainties and external disturbances, sliding mode control [12], [16], [17] and disturbance observer based approach [18] have been employed. However, a nominal model of the system is still needed.

The time-delayed control (TDC) has been originally introduced by [2] and [19] to handle fully actuated Euler-Lagrange systems with model uncertainties, unmodelled dynamics or unknown disturbances. The main idea of TDC is to make use of the continuity of the system dynamics and use the measurement information at the last time instant to approximately estimate all the unknown terms in the system at the current time instant. A constant gain matrix is introduced to replace the inertia matrix, so that the original highly nonlinear and strongly coupled dynamics of the Euler-Lagrange systems can be linearized and decoupled.

TDC has been widely applied in fully actuated systems but rarely in underactuated systems, because the stability and robustness rely on the system being fully actuated. In [20] and [21], the reduced order TDC has been introduced for underactuated systems. In [22] and [23], an adaptive control approach using time-delayed estimation (TDE) technique has been introduced to remove the restrictions of structural constraints and the requirement of a prior knowledge of some dynamic terms such as Coriolis and friction in the underactuated systems. Yet these approaches [20]–[23] involve many tuning parameters and make the controller design complicated. Moreover, the determination of the constant gain matrix still depends on the information of the inertia matrix.

In our previous publication [24], a data-driven TDC approach has been designed for fully actuated Euler-Lagrange systems. In this paper, the data-driven TDC technique will be extended and developed for underactuated Euler-Lagrange systems with unknown dynamics. It will be shown that, instead of controlling the states directly, sliding variables are defined to couple the actuated and unactuated states. The number of sliding variables is equal to the number of actuators. Therefore, a fully actuated form in terms of sliding variables can be obtained. Then, similar to the fully actuated systems, a new constant gain matrix can be directly obtained from measured input and output data. To deal with the TDE error, an adaptive sliding mode control is integrated in the data-driven design. The stability of the closed-loop system will be proven. The proposed algorithm involves only few controller parameters and the design effort is much lower than the other TDC approaches for underactuated systems.

Notation: $\mathbb{R}$, $\mathbb{R}_{\geq 0}$ and $\mathbb{R}_{>0}$ denote, respectively, the set of real numbers, non-negative real numbers and positive real numbers. $I_n \in \mathbb{R}^{n\times n}$ denotes an identity matrix. For a matrix Q, $Q \succ 0$ means that Q is positive definite. For a real number a, $|a|$ denotes the absolute value of a. For a vector x, $\|x\| = \sqrt{x^T x}$ is the Euclidean norm of x. For a matrix Q, $\lambda_{\min}(Q)$ denotes the minimal eigenvalue.

II. PRELIMINARIES

A. System description

Consider the underactuated Euler-Lagrange system

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + d_s = [\tau^T \ 0^T]^T \quad (1)$$

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where $q=[q^1\ q^2\cdots q^n]^T\in\mathbb{R}^n$ is the vector of generalized coordinates, $M(q)\in\mathbb{R}^{n\times n}$ is the positive definite inertia matrix, $C(q,\dot q)\in\mathbb{R}^{n\times n}$ is a matrix of centrifugal and Coriolis terms, $G(q)\in\mathbb{R}^n$ contains the gravitational terms, $F(\dot q)\in\mathbb{R}^n$ denotes viscous friction, $d_s\in\mathbb{R}^n$ describes the unknown disturbances and unmodelled dynamics, $\tau\in\mathbb{R}^m$ is the control input vector, where $(n-m)\le m<n$. The system (1) satisfies the following property.

Property 1 ([1]): The inertia matrix $M(q)$ is uniformly positive definite, i.e. there exist two constants $\mu_1,\mu_2\in\mathbb{R}_{>0}$ such that $\mu_1 I\le M(q)\le\mu_2 I$.

Let $x{=}[x_1^T\ x_2^T]^T{=}[q^T\ \dot q^T]$. Then (1) can be rewritten as

$$\dot x_1=x_2 \tag{2a}$$

$$\dot x_2=\psi(x)+\phi(x)u \tag{2b}$$

where $u{=}[u^1\ u^2\ \cdots u^m\ 0\cdots 0]^T{=}[\tau^T\ 0^T]^T$, $\phi(x){=}M^{-1}(q)$, $\psi(x){=}-M^{-1}(q)(C(q,\dot q)\dot q{+}G(q){+}F(\dot q){+}d_s)$ and $f(0){=}0$.

Without loss of generality, we make the following assumptions, which hold generally in practical systems.

Assumption 1: The control input is bounded, i.e. $u\in\mathbb{U}$. The boundedness is represented by a known constant $U_{\max}$, i.e. $|u^j|\le U_{\max}<\infty$, $\forall j=1,\cdots,n$.

Assumption 2: The states (e.g. the positions and the velocities) are bounded, i.e. $x\in\mathbb{X}$. For the Euler-Lagrange system, the states are constrained by $|q^j|\le q_{\max}<\infty$ and $|\dot q^j|\le\dot q_{\max}<\infty$, $\forall j=1,\cdots,n$.

Assumption 3: The reference signals of the positions q_r and the velocities $\dot q^r$ are bounded, i.e. $x\in\mathbb{X}$. For the Euler-Lagrange system, the references signals are constrained by $|q_r|{\le}q_{r,\max}{<}\infty$, $|\dot q_r|{\le}\dot q_{r,\max}{<}\infty$ and $|\ddot q_r|{\le}\ddot q_{r,\max}{<}\infty$.

Under Assumption 2, Assumption 4 can be made.

Assumption 4 ([1]): The function $\psi(x)$ in (2b) is locally Lipschitz in $x\in\mathbb{X}$. That means, given any $x_1\in\mathbb{X}$ and any $x_2\in\mathbb{X}$ in the neighbourhood of x_1, there exists always a positive bounded number $\alpha(x_1)$ that depends on x_1, so that $\|\psi(x_1)-\psi(x_2)\|\le\alpha(x_1)\|x_1-x_2\|$. Denote the maximum of $\alpha(x_1)$ over the set $\mathbb{X}$ by $K_1=\max_{x_1\in\mathbb{X}}\alpha(x_1)$.

At first, we rewrite system model (1) as

$$M(q)\ddot q+N(q,\dot q,d_s)=[\tau^T\ 0^T]^T \tag{3}$$

where $q=[q_a^T\ q_u^T]^T$ is composed of the actuated states $q_a\in\mathbb{R}^m$ and the unactuated states $q_u\in\mathbb{R}^{n-m}$, $M=\begin{bmatrix}M_{aa} & M_{au}\\ M_{au}^T & M_{uu}\end{bmatrix}$, $M_{aa}\in\mathbb{R}^{m\times m}$, $M_{au}\in\mathbb{R}^{m\times(n-m)}$, $M_{uu}\in\mathbb{R}^{(n-m)\times(n-m)}$, $N(q,\dot q,d_s)=C(q,\dot q)\dot q+G(q)+F(\dot q)+d_s=[N_a^T\ N_u^T]^T$, $N_a\in\mathbb{R}^m$, $N_u\in\mathbb{R}^{n-m}$.

The dynamics of the actuated and unactuated variables can be equivalently rewritten as

$$\ddot q_u=-M_{uu}^{-1}M_{au}^T\ddot q_a-M_{uu}^{-1}N_u \tag{4}$$

$$\ddot q_a=M_s^{-1}\tau+h_a \tag{5}$$

where $h_a=M_s^{-1}(M_{au}M_{uu}^{-1}N_u-N_a)$ and $M_s=M_{aa}-M_{au}M_{uu}^{-1}M_{au}^T$.

B. Control objective

Let $q_r=[q_{a,r}^T\ q_{u,r}^T]^T$, $e_a=q_a-q_{a,r}$, $e_u=q_u-q_{u,r}$ be the tracking error of actuated and unactuated states, respectively. Define coupled sliding variables as

$$S(t)=\Gamma_a S_a(t)+\Gamma_u S_u(t) \tag{6}$$

where $S_a{=}\dot e_a{+}\Upsilon_a e_a$ and $S_u{=}\dot e_u{+}\Upsilon_u e_u$ are the sliding variables of actuated and unactuated states in (1), respectively. $\Upsilon_a\in\mathbb{R}^{m\times m}$, $\Upsilon_u\in\mathbb{R}^{(n-m)\times(n-m)}$, $\Gamma_a\in\mathbb{R}^{m\times m}$ and $\Gamma_u\in\mathbb{R}^{m\times(n-m)}$ are constant matries that satisfy $\Upsilon_a{>}0$, $\Upsilon_u{>}0$, $\Gamma_a{>}0$ and $\Gamma_u{>}0$ are coupling parameters.

Using (4) and (5), the time derivative of (6) yields

$$\dot S=\Gamma_a\dot S_a+\Gamma_u\dot S_u=\ \Gamma_a(\ddot e_a+\Upsilon_a\dot e_a)+\Gamma_u(\ddot e_u+\Upsilon_u\dot e_u)$$
$$=f+S_r+g\tau \tag{7}$$

where $f=(\Gamma_a-\Gamma_u M_{uu}^{-1}M_{au}^T)h_a-\Gamma_u M_{uu}^{-1}N_u$, $S_r=\Gamma_a\Upsilon_a\dot e_a+\Gamma_u\Upsilon_u\dot e_u-\Gamma_a\ddot q_{a,r}-\Gamma_u\ddot q_{u,r}$ and $g=(\Gamma_a-\Gamma_u M_{uu}^{-1}M_{au}^T)M_s^{-1}$.

Consider the following controller designed by [22]

$$\tau{=}\bar g^{-1}(-\Lambda S{-}S_r{-}\tau_r),\ \ \tau_r{=}\begin{cases}\rho\frac{S}{\|S\|}, & \text{if } \|S\|\ge\epsilon_r\\ \rho\frac{S}{\epsilon}, & \text{if } \|S\|<\epsilon_r\end{cases} \tag{8}$$

where $\Lambda\in\mathbf{R}^{m\times m}$ satisfies $\Lambda>0$, τ_r deals with uncertainties using the gain ρ and ϵ is a small scalar to avoid chattering. A more detailed explanation of controller design in (8) can be found in [22]. Note that $\bar g$ is not a constant matrix but satisfies the following assumption.

Assumption 5 ([22]): A scalar E is known such that

$$\|g\bar g^{-1}-I_m\|\le E<1. \tag{9}$$

In [22] the closed-loop stability has been analyzed based on Assumption 5 and the uniformly ultimately boundedness of the closed-loop trajectories has been proven.

III. DATA-DRIVEN ADAPTIVE SLIDING MODE CONTROL

In (9) the mathematical expression of g is still required to select the matrix $\bar g$. Though a sufficiently large $\bar g$ may also satisfy Assumption 5, it may cause a weak or even unstable tracking performance, as shown by [25]. Therefore, $\bar g$ should be able to sufficiently represent the dynamics of g. In this section, we shall present a data-driven approach to determine $\bar g$ directly from the input and output data without any knowledge of g.

A. TDE-Based sliding variable

Let $\bar g_{\text{new}}{\in}\mathbb{R}^{m\times m}$ be a constant diagonal matrix. Then the nonlinear system equation (7) at time t can be rewritten as

$$\dot S(t)=H(t)+\bar g_{\text{new}}\tau(t) \tag{10}$$

where $H(t)=f(t)+S_r(t)+(g(t)-\bar g_{\text{new}})\tau(t)$. For a sufficient small sampling time T_s, the value of $H(t)$ can be approximated by its time-delayed value by considering the continuity of the function $H(t)$ [2], i.e. $H(t)\approx H(t-T_s)=\dot S(t-T_s)-\bar g_{\text{new}}\tau(t-T_s)$. Thus, one obtains

$$\dot S=\dot S_0+\bar g_{\text{new}}\Delta\tau+\epsilon(t). \tag{11}$$

where $\epsilon(t)=H(t)-H(t-T_s)$ denotes the TDE error. $\dot S_0=\dot S(t-T_s)$, $\tau_0=\tau(t-T_s)$ and $\Delta\tau=\tau(t)-\tau_0$.

B. Data-driven determination of the diagonal matrix $\bar g_{\text{new}}$

To get the diagonal matrix $\bar g_{\text{new}}$ based on data while guaranteeing the closed-loop stability, a persistently exciting input signal is used to excite the system. Choose $\bar g_{\text{new}}$ as

$$\bar g_{\text{new}}=\text{diag}\{\bar g^1,\bar g^2,\cdots,\bar g^m\}. \tag{12}$$

Then the system (11) can be rewritten as m subsystems by

$$\dot{S}^j = \dot{S}_0^j + \bar{g}^j_{\text{new}}\Delta\tau^j + \epsilon^j, \quad j = 1,\cdots,m. \tag{13}$$

where $S = [S^1 \;\; S^2 \;\; \cdots \;\; S^m]^T$.

Assume that the system is controllable. At sampling instants $t = kT_s,\ k = 1,2,\cdots,N$, a persistently exciting control input sequence $\tau^j(kT_s)$ satisfying Assumption 1 is used to excite the system to get the dynamic behaviour of the system. In practice, some commonly used test signals such as pseudo random binary sequence can be applied here.

The measurements of the states $q_a^j(kT_s)$, $\dot{q}_a^j(kT_s)$, $q_u^j(kT_s)$ and $\dot{q}_u^j(kT_s)$ are collected. Note that $\ddot{q}^j(kT_s)$ can be calculated by the Euler method as $\ddot{q}^j(kT_s) = \frac{\dot{q}^j(kT_s) - \dot{q}^j((k-1)T_s)}{T_s}$.

Under Assumption 1 and recalling Property 1, the following Lemma can be obtained.

Lemma 1: Given system (1) with Property 1 and choose a sufficiently small sampling time T_s. Under Assumptions 1, there always exists a finite positive number K_1, so that the function $\ddot{q}$ satisfies the following relation

$$\|\ddot{q} - \ddot{q}_0\| \leq K_1\|x - x_0\| + 4\mu_1^{-1}U_{\max}, \tag{14}$$

where x_0 and $\ddot{q}_0$ denote, respectively, the value of x and $\ddot{q}$ at the last sampling instant.

Proof: Let x_0 and u_0 denote, respectively, the value of x and u at the last sampling instant. Due to Property 1, the matrix $\phi(x) = M^{-1}(q)$ is bounded by $\mu_2^{-1}I \leq \phi(x) \leq \mu_1^{-1}I$. Under Assumption 1, we obtain

$$\begin{aligned}
&\|\ddot{q} - \ddot{q}_0\| = \|\psi(x) + \phi(x)u - \psi(x_0) - \phi(x_0)u_0\| \\
&\leq K_1\|x - x_0\| + \|\phi(x)u - \phi(x)u_0 + \phi(x)u_0 - \phi(x_0)u_0\| \\
&\leq K_1\|x - x_0\| + \mu_1^{-1}\|u - u_0\| + 2\mu_1^{-1}\|u_0\| \\
&\leq K_1\|x - x_0\| + 4\mu_1^{-1}U_{\max}.
\end{aligned} \tag{15}$$

Thus, the inequality (14) holds. ■

Making use of Lemma 1, the following lemma shows the boundedness of the sliding mode variables $\dot{S} - \dot{S}_0$.

Lemma 2: Given system (1) with Property 1 and choose a sufficiently small sampling time T_s. Under Assumptions 1-4, there always exists finite positive numbers K and K_r, so that the function $\dot{S} - \dot{S}_0$ satisfies the relation $\|\dot{S} - \dot{S}_0\| \leq KU_{\max} + K_rT_s$, where $S_0 = S(t - T_s)$.

Proof: From (7), we obtain $\|\dot{S} - \dot{S}_0\| = \|\Gamma_a\ddot{e}_a + \Gamma_a\Upsilon_a\dot{e}_a + \Gamma_u\ddot{e}_u + \Gamma_u\Upsilon_u\dot{e}_u - \Gamma_a\ddot{e}_{a,0} - \Gamma_a\Upsilon_a\dot{e}_{a,0} - \Gamma_u\ddot{e}_{u,0} - \Gamma_u\Upsilon_u\dot{e}_{u,0}\| \leq [\Gamma_a \;\; \Gamma_u]\|\ddot{e} - \ddot{e}_0\| + [\Gamma_a\Upsilon_a \;\; \Gamma_u\Upsilon_u]\|\dot{e} - \dot{e}_0\| \leq [\Gamma_a \;\; \Gamma_u]\|\ddot{q} - \ddot{q}_0\| + [\Gamma_a\Upsilon_a \;\; \Gamma_u\Upsilon_u]\|\dot{q} - \dot{q}_0\| + [\Gamma_a \;\; \Gamma_u]\|\ddot{q}_r - \ddot{q}_{r,0}\| + [\Gamma_a\Upsilon_a \;\; \Gamma_u\Upsilon_u]\|\dot{q}_r - \dot{q}_{r,0}\|$. Under Assumption 3, it is reasonable to assume that there exists a constant $K_r > 0$ such that $[\Gamma_a \;\; \Gamma_u]\|\ddot{q}_r - \ddot{q}_{r,0}\| + [\Gamma_a\Upsilon_a \;\; \Gamma_u\Upsilon_u]\|\dot{q}_r - \dot{q}_{r,0}\| \leq K_rT_s$. Making use of Lemma 1 and considering that $\|\dot{q} - \dot{q}_0\| \leq \|x - x_0\|$, we obtain

$$\begin{aligned}
&\|\dot{S} - \dot{S}_0\| \leq [\Gamma_a \;\; \Gamma_u]\|\ddot{q} - \ddot{q}_0\| + [\Gamma_a\Upsilon_a \;\; \Gamma_u\Upsilon_u]\|\dot{q} - \dot{q}_0\| + K_rT_s \\
&\leq [\Gamma_a \;\; \Gamma_u](K_1\|x - x_0\| + 2\mu_1^{-1}U_{\max}) \\
&\quad + [\Gamma_a\Upsilon_a \;\; \Gamma_u\Upsilon_u]\|\dot{q} - \dot{q}_0\| + K_rT_s \\
&= [\Gamma_aK_1 \;\; \Gamma_uK_1]\|x - x_0\| + [2\Gamma_a\mu_1^{-1} \;\; 2\Gamma_u\mu_1^{-1}]U_{\max} \\
&\quad + [\Gamma_a\Upsilon_a \;\; \Gamma_u\Upsilon_u]\|x - x_0\| + K_rT_s.
\end{aligned} \tag{16}$$

Many studies [26]–[28] assume that the actuator's dynamics are much faster than the dynamics of the system itself. Thus, the change in the position $x_1 - x_{1,0}$ and the change in the velocity $x_2 - x_{2,0}$ are much smaller than the change in input $u - u_0$, i.e. $\|x - x_0\| \ll \|u - u_0\|$.

Then, from (16) we obtain $\|\dot{S} - \dot{S}_0\| \leq [\Gamma_aK_1 \;\; \Gamma_uK_1]\|u - u_0\| + [2\Gamma_a\mu_1^{-1} \;\; 2\Gamma_u\mu_1^{-1}]U_{\max} + [\Gamma_a\Upsilon_a \;\; \Gamma_u\Upsilon_u]\|u - u_0\| + K_rT_s \leq [2\Gamma_a(K_1 + \mu_1^{-1} + \Upsilon_a) \;\; 2\Gamma_u(K_1 + \mu_1^{-1} + \Upsilon_u)]U_{\max} + K_rT_s \leq KU_{\max} + K_rT_s$, where $K = [2\Gamma_a(K_1 + \mu_1^{-1} + \Upsilon_a) \;\; 2\Gamma_u(K_1 + \mu_1^{-1} + \Upsilon_u)]$. ■

Assumption 6: For a sufficiently large number of data samples N in offline data set and under a persistently exciting control input signal τ, the largest values

$$\begin{aligned}
\Delta\dot{S}^j_{\max,N} &= \max_k\{|\dot{S}^j(kT_s) - \dot{S}^j((k-1)T_s)|\} \\
\Delta\tau^j_{\max,N} &= \max_k\{|\tau^j(kT_s) - \tau^j((k-1)T_s)|\}
\end{aligned} \tag{17}$$

provide a good estimation of the largest acceleration and the largest input change in a bounded moving area.

Under Assumption 6 and making use of Lemma 2, the following lemma shows that the TDE error ϵ is bounded if $\bar{g}^1, \bar{g}^2, \cdots, \bar{g}^m$ are selected suitably.

Lemma 3: There exists a constant $\bar{\epsilon} \in \mathbb{R}_{>0}$ such that $\|\epsilon\| \leq \bar{\epsilon}$, if the sampling period T_s is sufficiently small and $\bar{g}^1, \bar{g}^2, \cdots, \bar{g}^m$ in (12) are chosen as

$$\bar{g}^j = \Delta\dot{S}^j_{\max,N}/\Delta\tau^j_{\max,N}, \quad j = 1,2,\cdots,m. \tag{18}$$

Proof: Under Assumption 4 and considering (13), the TDE error $\epsilon^j,\ j = 1,2,\cdots,m$ of j-th sliding variable is

$$|\epsilon^j(kT_s)| = |\dot{S}^j(kT_s) - \dot{S}^j((k-1)T_s) - \bar{g}^j\Delta\tau^j(kT_s)| \tag{19}$$

Taking into account (17) and (18), we obtain $|\epsilon^j(kT_s)| = |\dot{S}^j(kT_s) - \dot{S}^j((k-1)T_s) - \frac{\Delta\dot{S}^j_{\max,N}}{\Delta\tau^j_{\max,N}}\Delta\tau^j(kT_s)| = \Delta\dot{S}^j_{\max,N}\left|\frac{\dot{S}^j(kT_s) - \dot{S}^j((k-1)T_s)}{\Delta\dot{S}^j_{\max,N}} - \frac{\Delta\tau^j(kT_s)}{\Delta\tau^j_{\max,N}}\right| \leq 2\Delta\dot{S}^j_{\max,N}$.

According to Lemma 2, $|\epsilon^j(kT_s)| \leq 2\Delta\dot{S}^j_{\max,N} = 2\max_k\{|\dot{S}^j(kT_s) - \dot{S}^j((k-1)T_s)|\} \leq 2\max_k\{\|\dot{S}(kT_s) - \dot{S}((k-1)T_s)\|\} \leq 2KU_{\max} + 2K_rT_s$. Therefore, $\|\epsilon(kT_s)\| = \sqrt{\sum_{j=1}^n(\epsilon^j(kT_s))^2} \leq \bar{\epsilon}$, where $\bar{\epsilon} = \sqrt{n}\cdot 2K(KU_{\max} + K_rT_s)$. Thus, the conclusion of Lemma 3 holds. ■

According to Lemma 3, $\bar{g}_{\text{new}}$ can be selected as

$$\bar{g}_{\text{new}} = \text{diag}\left\{\frac{\Delta\dot{S}^1_{\max,N}}{\Delta\tau^1_{\max,N}}, \cdots, \frac{\Delta\dot{S}^m_{\max,N}}{\Delta\tau^m_{\max,N}}\right\}. \tag{20}$$

Correspondingly, the unknown TDE error ϵ is bounded.

C. Data-driven adaptive control

Consider the following controller

$$\begin{aligned}
&\tau(kT_s) = \tau((k-1)T_s) + \Delta\tau, \\
&\Delta\tau = \bar{g}^{-1}_{\text{new}}(-\Lambda S - \dot{S}_0 - \Delta u_r),\ \Delta u_r = K_s \cdot sgn(S)
\end{aligned} \tag{21}$$

where $S = \Gamma_a\dot{e}_a + \Gamma_a\Upsilon_ae_a + \Gamma_u\dot{e}_u + \Gamma_u\Upsilon_ue_u$ is the sliding variable defined in (6). To tackle the bounded TDE error ϵ, an adaptive sliding mode controller Δu_r is introduced with the gain $K_s(t)$ defined as [29]

$$\dot{K}_s = \begin{cases} \bar{K}_s \cdot \|S\| \cdot sgn(\|S\| - \epsilon_b) & \text{if } K_s \geq \mu \\ \mu & \text{if } K_s < \mu \end{cases} \tag{22}$$

with $K_s(0) > \mu,\ \bar{K}_s > 0,\ \epsilon_b > 0,\ \mu > 0$ and sgn denotes the sign function. The parameters $\epsilon_b > 0$ and μ are very small and μ is introduced in order to get only positive values for K_s. Next, we suppose that $K_s(t) > \mu$ for all $t > 0$ [29].

Using Lemma 3, the following Lemma shows that the gain K_s in Δu_r in (21) will not increase to be infinity large.

Lemma 4: Given system (1) with Property 1 and choose a sufficiently small sampling time T_s with the sliding variable S defined in (7) controlled by (21), the gain $K_s(t)$ has an upper-bound, i.e. there exists a positive constant K_s^* so that

$$K_s(t) < K_s^*, \quad \forall t > 0 \tag{23}$$

Proof: Substituting (21) into (11) yields

$$\dot{S} = -\Lambda S - K_s sgn(S) + \epsilon \tag{24}$$

Suppose that the initial value $|S(t_0)| > \epsilon_b$. From (22), K_s increases and there exists a time instant t_1 such that $\dot{S}_r(t_1) = 0$ and $K_s(t_1) = \| -\Lambda S(t_1) + \epsilon(t_1)\|$. From $t = t_1$, the gain K_s is large enough to make the sliding variable S decrease. Thus, in a finite time t_2, $\|S\| < \epsilon_b$. It yields that the gain K_s reaches a maximum value at $t = t_2$ and decreases after $t = t_2$. Then, there exists a time instant $t_3 > t_2$ such that $\dot{S}_r(t_3) = 0$ and $K_s(t_3) = \| -\Lambda S(t_3) + \epsilon(t_3)\|$. From $t = t_3$, the gain K_s is not large enough to deal with the TDE error ϵ as K_s is decreasing. It yields that there exists a time instant $t_4 > t_3$ such that $\|S(t_4)\| < \epsilon_b$. Then, the process restarts from the beginning. By using Assumption 2 and Lemma 3, the sliding variable S and the TDE error ϵ are bounded. Thus, the gain $K_s(t)$ is bounded uniformly on t by $K_s^* > 0$. ∎

In the next, the error dynamics of the unactuated states will be analyzed. Rewrite (4) by

$$\ddot{q}_a = -(M_{au}^T)^{-1} M_{uu}\ddot{q}_u - (M_{au}^T)^{-1} N_u \tag{25}$$

Substituting (25) into (7) yields

$$\begin{aligned}\dot{S} &= \Gamma_a \ddot{e}_a + \Gamma_a \Upsilon_a \dot{e}_a + \Gamma_u \ddot{e}_u + \Gamma_u \Upsilon_u \dot{e}_u \\ &= \Gamma_a(-(M_{au}^T)^{-1} M_{uu}\ddot{e}_u - (M_{au}^T)^{-1} N_u) \\ &\quad + \Gamma_a \Upsilon_a \dot{e}_a + \Gamma_u \ddot{e}_u + \Gamma_u \Upsilon_u \dot{e}_u\end{aligned} \tag{26}$$

By substituting (21) and (26) into (11), we obtain $\Gamma_a(-(M_{au}^T)^{-1} M_{uu}\ddot{e}_u - (M_{au}^T)^{-1} N_u) + \Gamma_a \Upsilon_a \dot{e}_a + \Gamma_u \ddot{e}_u + \Gamma_u \Upsilon_u \dot{e}_u = -\Lambda(\Gamma_a \dot{e}_a + \Gamma_a \Upsilon_a e_a + \Gamma_u \dot{e}_u + \Gamma_u \Upsilon_u e_u) + \epsilon - \Delta u_r$. Let $x_{u,1} = e_u$ and $x_{u,2} = \dot{e}_u$. Then

$$\begin{aligned}\dot{x}_{u,1} &= x_2 \\ \dot{x}_{u,2} &= -k_{u,1}x_{u,1} - k_{u,2}x_{u,2} + b^{-1}(\phi_u - \Delta u_r)\end{aligned} \tag{27}$$

where $\phi_u = \Gamma_a(M_{au}^T)^{-1}N_u - \Gamma_a \Upsilon_a \dot{e}_a - \Lambda\Gamma_a \dot{e}_a - \Lambda\Gamma_a\Upsilon_a e_a + \epsilon$, $k_{u,1} = \frac{\Lambda\Gamma_u\Upsilon_u}{b}$, $k_{u,2} = \frac{\Gamma_u\Upsilon_u + \Lambda\Gamma_u}{b}$, $b = \Gamma_u - \Gamma_a(M_{au}^T)^{-1}M_{uu}$.

One can design the Γ_a amd Γ_u such that $b > 0$ holds. Therefore, $k_{u,1} > 0$ and $k_{u,2} > 0$ hold. Let $x_u = \begin{bmatrix} x_{u,1} \\ x_{u,2}\end{bmatrix}, A_u = \begin{bmatrix} 0 & I \\ -k_{u,1} & -k_{u,2}\end{bmatrix}, B_u = \begin{bmatrix} 0 \\ b^{-1}\end{bmatrix}$. Then (27) can be rewritten as

$$\dot{x}_u = A_u x_u + B_u(\phi_u - \Delta u_r) \tag{28}$$

Note that A_u is Hurwitz and ϕ_u is a function including system state q, reference signal q_r and TDE error ϵ. From Assumption 2 and Assumption 3, both the true states and the reference signals are bounded. From Lemma 3, ϵ is bounded. Therefore, there exists a constant $\bar{\phi}_u$ such that $\phi_u \le \bar{\phi}_u$.

D. Stability analysis of the data-driven adaptive control

In this subsection, it will be shown that the tracking error is uniformly ultimately bounded (UUB) [30]–[32].

Theorem 1: Given the plant (1) controlled by (21) where $\bar{g}_{new}$ is obtained by (18). Under Properties 1 and Assumption 1-4, the tracking error of the closed-loop system is UUB.

Proof: Consider the Lyapunov function candidate

$$V = \frac{1}{2}S^2 + \frac{1}{2\gamma}(K_s - K_s^*)^2 + \frac{1}{2}x_u^2. \tag{29}$$

where $\gamma > 0$ is a positive scalar factor. As discussed in Subsection C of Section III, one supposes that $K_s(t) > \mu$ for all $t > 0$. Thus, we only discuss when $K_s > \mu$ in the following.

At first we show that the Lyapunov candidate function V defined in (29) is bounded by two continuous, strictly increasing functions. Let $\gamma_1(\|x_u\|) = c_1 S^2 + \frac{c_1}{\gamma}(K_s - K_s^*)^2 + c_1 x_u^2$ and $\gamma_2(\|x_u\|) = c_2 S^2 + \frac{c_2}{\gamma}(K_s - K_s^*)^2 + c_2 x_u^2$, where $c_1 \in \mathbb{R}_{\ge 0}$, $c_2 \in \mathbb{R}_{\ge 0}$ and $c_1 \le \frac{1}{2} \le c_2$. Considering (29), for the function V it holds $\gamma_1(\|x_u\|) \le V \le \gamma_2(\|x_u\|)$.

Next, we show that the function V is decreasing outside a compact set of x_u. From (21), (24) and (28), we obtain $\dot{V} = S(-\Lambda S + \epsilon - K_s \cdot sgn(S)) + \frac{1}{\gamma}(K_s - K_s^*) \cdot \bar{K}_s \cdot \|S\| sgn(\|S\| - \epsilon_b) + x_u^T(A_u x_u + B_u(-K_s sgn(S) + \phi_u))$.

Because A_u is Hurwitz, there exists a positive definite matrix P such that $x_u^T A x_u \le -x_u^T P x_u$ holds. According to Lemma 3, $\|\epsilon\| \le \bar{\epsilon}$. One obtains $\dot{V} \le (-\Lambda S + \epsilon)\|S\| - K_s\|S\| + \frac{(K_s - K_s^*)\bar{K}_s\|S\| sgn(\|S\| - \epsilon_b)}{\gamma} - x_u^T P x_u + \|x_u^T\| b^{-1}(K_s^* + \bar{\phi}_u) = -\|S\|(\Lambda S - \bar{\epsilon} + K_s^*) + (K_s - K_s^*)(-\|S\| + \frac{1}{\gamma}\bar{K}_s\|S\| sgn(\|S\| - \epsilon_b)) - x_u^T P x_u + \|x_u^T\| b^{-1}(K_s^* + \bar{\phi}_u))$.

From Lemma 4, there always exists $K_s^* > 0$ such that $K_s(t) - K_s^* < 0$ for all $t > 0$. It yields

$$\begin{aligned}\dot{V} = &- \|S\|(\Lambda S - \bar{\epsilon} + K_s^*) - x_u^T P x_u \\ &- \|K_s - K_s^*\|\Big\{ - \|S\| + \frac{1}{\gamma}\bar{K}_s\|S\| sgn(\|S\| - \epsilon_b) \\ &- \frac{\|x_u^T\| b^{-1}(K_s^* + \bar{\phi}_u)}{\|K_s - K_s^*\|}\Big\}\end{aligned} \tag{30}$$

Let $\xi_1 = \Lambda S - \bar{\epsilon} + K_s^*$ and $\xi_2 = -\|S\| + \frac{1}{\gamma}\bar{K}_s\|S\| sgn(\|S\| - \epsilon_b) - \frac{\|x_u^T\| b^{-1}(K_s^* + \bar{\phi}_u)}{\|K_s - K_s^*\|}$. Then, from (30) one gets

$$\dot{V} = -\|S\|\xi_1 - x_u^T P x_u - \|K_s - K_s^*\|\xi_2. \tag{31}$$

From Lemma 4 and (23), $\xi_1 > 0$ holds.

In the next, we analyze the stability by considering two cases. In the first case, $\|S\| \ge \epsilon_b$. In the second case, $\|S\| < \epsilon_b$.

Case 1: Suppose that $\|S\| \ge \epsilon_b$. ξ_2 is positive if $-\|S\| + \frac{1}{\gamma}\bar{K}_s\|S\| sgn(\|S\| - \epsilon_b) - \frac{\|x_u^T\| b^{-1}(K_s^* + \bar{\phi}_u)}{\|K_s - K_s^*\|} > 0$. Thus, $\xi_2 > 0$ holds if

$$\gamma < \frac{\bar{K}_s \epsilon_b \|K_s - K_s^*\|}{\epsilon_b\|K_s - K_s^*\| + \|x_u^T\| b^{-1}(K_s^* + \bar{\phi}_u)} \tag{32}$$

From Assumption 2 and Assumption 3, $\|x_u\|$ is bounded. Thus, it is always possible to choose γ such that (32) holds. From (31), one get $\dot{V} < 0$. Therefore, finite time convergence to a domain $\|S\| \le \epsilon_b$ is guaranteed from any initial condition $\|S(0)\| > \epsilon_b$.

Case 2: Suppose that $\|S\| < \epsilon_b$. Then ξ_2 can be negative. So $\dot{V}$ could be sign indefinite. Therefore, it is necessary to analyze $\dot{V}$ in terms of x_u in the domain $\|S\| < \epsilon_b$.

In the following, we consider the influence of unactuated state x_u on $\dot{V}$ when $\|S\| < \epsilon_b$. Let $p = \lambda_{\min}(P)$. Because P is a positive definite matrix, there is $p > 0$ and $x_u^T P x_u \ge$

$px_u^T x_u = p\|x_u^T\|\|x_u\|$. From (30) one obtains

$$\dot{V} \le -\|S\|(\Lambda S - \bar{\epsilon} + K_s^*) - \|K_s - K_s^*\|\big(-\|S\| - \frac{1}{\gamma}\bar{K}_s\|S\| + \frac{p\|x_u^T\|\|x_u\| - \|x_u^T\|b^{-1}(K_s^* + \bar{\phi}_u)}{\|K_s - K_s^*\|}\big) \quad (33)$$

Let $\xi_4 = -\|S\| - \frac{1}{\gamma}\bar{K}_s\|S\| + \frac{p\|x_u^T\|\|x_u\| - \|x_u^T\|b^{-1}(K_s^* + \bar{\phi}_u)}{\|K_s - K_s^*\|}$. Then, from (33) one gets

$$\dot{V} = -\|S\|\xi_1 - \|K_s - K_s^*\|\xi_4. \quad (34)$$

If ξ_4 is positive, $\dot{V}$ is negative. $\xi_4 > 0$ holds, when

$$k_a\|x_u\|^2 - k_b\|x_u\| - k_c > 0 \quad (35)$$

where $k_a = p > 0$, $k_b = b^{-1}(K_s^* + \bar{\phi}_u) > 0$ and $k_c = \|K_s - K_s^*\|\|S\|(1 + \frac{1}{\gamma}\bar{K}_s) > 0$. Therefore, from (34) and (35), $\dot{V} < 0$, when $\|x_u\| > \frac{k_b + \sqrt{k_b^2 + 4k_a k_c}}{2k_a}$. Hence, $\dot{V}$ is negative outside of the compact set $\{\|x_u\| \le \frac{k_b + \sqrt{k_b^2 + 4k_a k_c}}{2k_a}\}$.

Combining Case 1 and Case 2, the following conclusion can be obtained. When $\|S\| \ge \epsilon_b$, $\dot{V}$ is negative and V decreases. In the region $\|S\| < \epsilon_b$, $\dot{V}$ is negative and V still decreases when $\|x_u\| \ge \frac{k_b + \sqrt{k_b^2 + 4k_a k_c}}{2k_a}$. UUB of x_u can be concluded [30], [31], which implies that $S, e_u, \dot{e}_u, K_s$ are bounded. From (6), one gets

$$\dot{e}_a = -\Upsilon_a e_a - \Gamma_a^{-1}(\Gamma_u \dot{e}_u + \Gamma_u \Upsilon_u e_u) + \Gamma_a^{-1} S \quad (36)$$

Because $\Gamma_a > 0$, Γ_a^{-1} exists. Considering $\Upsilon_a > 0$ and $S, e_u, \dot{e}_u, K_s$ being bounded, one concludes that $e_a, \dot{e}_a$ are bounded. Therefore, the conclusion of Theorem 1 holds. ■

IV. SIMULATION EXAMPLE

In this section, the offshore boom crane [22] is used to evaluate the proposed data-driven approach. In Fig. 1, $\{OX_EY_E\}$ and $\{OX_SY_S\}$ define, respectively, the Earth-fixed and ship-fixed coordinates. ϑ is the luffing angle of the boom, α is the swing with respect to Y_s of the payload having mass m_p, χ is the roll angle of the ship, $L(t)$ is the length of the rope, P_L, m and J are, respectively, the length, mass and inertia of the boom and the point O. The system dynamics can be described by [22], [33] as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d_s = [\tau^T \quad 0]^T. \quad (37)$$

where $\tau = [\tau_1 \quad \tau_2]^T, q = [q_1 \quad q_2 \quad q_3]^T = [\vartheta - \Psi \quad L \quad \alpha - \chi]^T$,

$$M(q) = \begin{bmatrix} J + m_p P_L^2 & -m_p P_L C_{1-3} & -m_p P_L q_2 S_{1-3} \\ -m_p P_L C_{1-3} & m_p & 0 \\ -m_p P_L q_2 S_{1-3} & 0 & m_p q_2^2 \end{bmatrix}$$

$S_{1-3} = sin(q_1 - q_3), \quad C_{1-3} = cos(q_1 - q_3)$,

the centrifugal and Coriolis terms $C(q,\dot{q})$ and the gravitational terms $G(q)$ can be found in [22], [33]. The true system parameters used in the simulations are $m = 20\ kg$, $m_p = 0.5\ kg$, $d = 0.4\ m$ and $J = 6.5 kg\cdot\ m^2$. In the simulation, $d_s = (0.1sin(0.01t) + d_n)[1 \quad 1 \quad 1]^T$, with d_n a zero-mean Gaussian noise with variance 0.002.

The proposed data-driven adaptive controller is applied to the offshore boom crane without using any model knowledge. We only know the offshore boom crane belongs to Euler-Lagrange system and the number of actuated states and unactuated states. Select $\Gamma_a = diag\{50, 50\}$, $\Gamma_u = 50[1 \quad 1]^T$, $\Upsilon_a = 8$ and $\Upsilon_u = 8$ to generate the coupled sliding variable defined in (6). The matlab function *idinput*

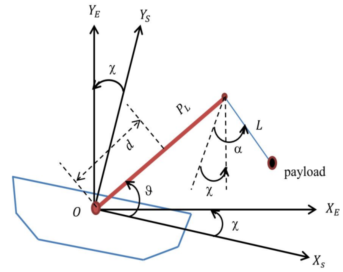

Fig. 1: Schematic description of offshore boom crane [22]

is used to generate a persistently exciting input torque of each joint $u^j(kT_s)$, $k = 1, 2, \cdots, N$; $j = 1, 2$. Let $N = 2^{13} = 8192$. The position q^j and the velocity $\dot{q}^j$ are measured at discrete time instants kT_s, $k = 1, \cdots, N$. The sampling time $T_s = 0.001$s. According to the coupled sliding variable given in (6) with parameters $\Gamma_a = diag\{50, 50\}$, $\Gamma_u = 50[1 \quad 1]^T$, $\Upsilon_a = 8$ and $\Upsilon_u = 8$, the first derivative of the coupled sliding variable $\dot{S}(kT_s)$, $k = 1, 2, \cdots, N$ is calculated by the Euler method as $\dot{S}(kT_s) = \frac{S(kT_s) - S((k-1)T_s)}{T_s}$. Among these 8192 data samples, the largest value $\Delta\dot{S}_{\max}^j$ and $\Delta u_{\max}^j$ are obtained by (17), respectively, as $\Delta\dot{S}_{\max,N} = [68.23 \quad 267.75]^T$ and $\Delta u_{\max,N} = [2 \quad 2]^T$. Then the constant gain matrix $\bar{g}_{new}$ is obtained from (20) as $\bar{g}_{new} = diag\{34.12, 133.87\}$. Select $\Lambda = diag\{15, 15\}$, $\bar{K}_s = 2$, $\epsilon_b = 0.5$, $\mu = 0.5$. The controller is constructed as (21).

For comparison, the adaptive robust control (ARC) approach [22] is applied. As shown in [22], the gain matrix $\bar{g} = (\Gamma_a - \Gamma_u M_{uu}^{-1} M_{au}^T) M_s^{-1}$ is not constant but designed based on the knowledge of $M(q)$ with the nominal parameters $\hat{m}_p = 0.45\ kg$ and $\hat{J} = 6\ kg{\cdot}m^2$. The parameters $\Gamma_a = diag\{50, 50\}$, $\Gamma_u = 50[1 \quad 1]^T$, $\Upsilon_a = 8$, $\Upsilon_u = 8$ and $\Lambda = diag\{15, 15\}$ are chosen to be the same as in the data-driven adaptive controller. The other parameters used in the ARC approach can be found in [22]. The objective of transporting the payload to (a_L, b_L) can be transformed into stabilization around the position $q_1^r = arccos(a_L/P_L)$, $q_2^r = \sqrt{P_L^2 - a_L^2} - b_L$, $q_3^r = 0$. In the simulation, we take $a_L = 0.4\ m$, $b_L = 0.2\ m$ and $P_L = 0.8\ m$, resulting in $q_r^1 = 1.05\ rad$ (i.e. 60 *degrees*), $q_r^2 = 0.5\ m$ and $q_r^3 = 0$. The initial states of the system are chosen as $q(0) = [0.2 \quad 0.1 \quad 0.1]^T$.

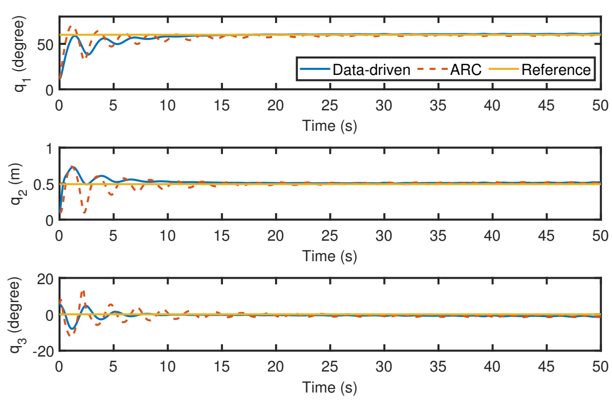

Fig. 2: Tracking Performance

As can be seen from Figure 2 and Figure 3, the data-driven adaptive controller has reduced the overshoot of the tracking

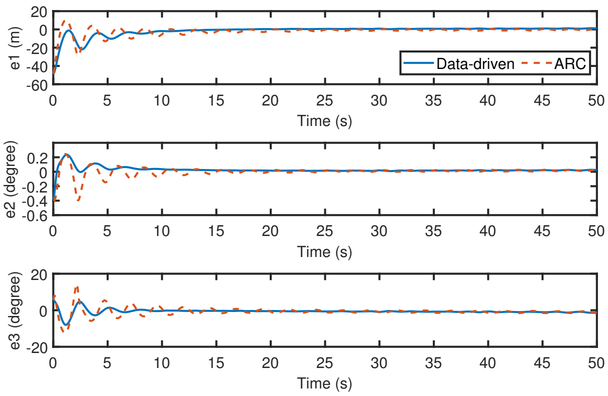

Fig. 3: Tracking Error

performance in q_1 and q_3 achieved by the ARC controller. The oscillated performance in q_2 of the ARC approach has also been improved by the data-driven adaptive controller. These results show that the data-driven adaptive controller achieves a better tracking performance.

V. CONCLUSIONS

A data-driven adaptive control approach is developed for unknown underactuated Euler-Lagrange systems. Compared with the existing TDC approaches, the proposed control approach only uses the input and output data without using any knowledge of the inertia matrix. Moreover, only few control parameters are required. In the future, the data-driven optimal controller will be investigated to achieve the optimal performance for underactuated systems.

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