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Robust iterative learning control design for a class of uncertain batch processes

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***Abstract*— This paper investigates the problem of designing robust iterative learning control laws for discrete-time batch processes with norm-bounded parameter uncertainties. A law of proportional-differential type is designed to achieve robust convergence of the tracking error in the batch-to-batch direction. It is shown that the design problem can be written as a two-dimensional system. Then, the recently developed non-conservative conditions for (structural) stability analysis for a linear Roesser model are used. The conditions for the existence and computation of the required control law matrices are linear matrix inequality-based. Finally, comparative simulation results show the effectiveness of the new design.**

I. Introduction

Iterative learning control (ILC) applies to systems or processes that perform the same finite-duration task repeatedly. The aim is to improve the accuracy by using data from previous executions to update the control action for the next one. Early literature is covered in, e.g., [1], [2].

A literature review indicates that ILC has attracted considerable research attention since it has been used for improving the control performance in many practical problems such as industrial robotics, see, e.g., [3] and wafer stage motion systems, see, for example, [4]. Additionally, see, e.g., [5], [6], ILC can be directly applied in the chemical process industries and, in particular, batch processes.

The application to industrial batch processes is, in the main, based on the lifting technique for discrete-time ILC [1], [2] where a common problem is how to design for batch domain convergence. However, robustness analysis concentrated on the models obtained with the lifting technique may fail to work in practical applications because of the drawback that the tracking error may grow quite large in the early stages of learning. Thus, the feedback controller is commonly employed along with the ILC law to ensure closed-loop stability. In this situation, the ILC laws designed for robustness against the original plant uncertainty may fail to compensate for the uncertainty of the controlled system.

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An alternative approach to ILC design is to use the two-dimensional (2D) structure of the dynamics [7], [8]. This approach allows us to consider the interaction between the batch-to-batch error and transient response along the batches. Unfortunately, a direct application of 2D system models and their stability conditions are computationally cumbersome; sufficient but not necessary conditions are often required to compute the control law, see [7].

This paper aims to use some known less conservative stability and stabilization conditions for 2D Roesser models [9], [10], [11]. These results lead to linear matrix inequality (LMI)-based conditions for ILC control law design applied to discrete-time linear batch processes. In particular, in this paper, procedures for ILC law are developed using the fact that the structural stability of the 2D Roesser model imposes tracking error convergence of the resulting ILC law. This paper extends the considerations presented in [12], where the focus was on developing LMI-based design conditions for PD-type ILC laws for nominal linear batch processes, by taking into account the existence of a process with norm-bounded parameter uncertainties. A similar problem was also considered by the authors in [13], but with the application of linear repetitive process stability theory. The new and less conservative conditions are established by utilization of the results on the stability of a Roesser model with state feedback to guarantee robust stability of the resulting ILC system along both the time and batch directions.

The new design can reduce conservatism and hence improve the applicability of developed results. A numerical example illustrates the approach's benefits and demonstrates that the new LMI conditions are less conservative than currently available. Also, the tracking performance of the controlled dynamics is compared with some known results to indicate the potential interest in this paper's outcomes.

Throughout this paper, the null and identity matrices with compatible dimensions are denoted by 0 and I, respectively, and the notation $[\,]_{n,0}$ (respectively $[\,]_{0,n}$) denotes an empty matrix with n rows and 0 columns (respectively n columns and 0 rows). For a real matrix P, the notation of $P \succ 0$ ($P \prec 0$) means that $P = P^T$ and that P is positive (negative) definite. In addition, $\rho(P)$ denotes the spectral radius of P. Furthermore $\mathrm{sym}\{\mathrm{P}\}$ denotes $P + P^*$ where P^* is the transpose conjugate of a matrix P, $\overline{\mathbb{C}}$ denotes $\mathbb{C} \cup \{\infty\}$ where $\mathbb{C}$ is the set of complex numbers. Finally, for a matrix S and block matrix $\hat{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ with compatible dimensions, $S \star \hat{S} = S_{22} + S_{21}S(I - S_{11}S)^{-1}S_{12}$ denotes for the linear fractional transformation (LF) of $\hat{S}$ with respect to S, and $\otimes$ denotes the matrix Kronecker product.

The proofs of the results in this paper make use of the following result.

Lemma 1: [14] Given matrices $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z} = \mathcal{Z}^T$, $\delta(t)$ of compatible dimensions, then

$$\mathcal{Z} + \mathrm{sym}\{\mathcal{X}\delta(t)\mathcal{Y}\} \prec 0$$

for all $\delta(t)$ satisfying $\delta^T(t)\delta(t) \preceq I$ if and only if there exists a positive scalar ε such that

$$\mathcal{Z} + \varepsilon\mathcal{X}\mathcal{X}^T + \varepsilon^{-1}\mathcal{Y}^T\mathcal{Y} \prec 0.$$

II. Preliminaries

Let $t \in [0, N-1]$ be the discrete-time index where N is the fixed number of time steps for each batch and $k \geq 1$ is the batch number. Then, the class of uncertain discrete-time batch processes considered over this interval is described by the state-space model

$$\begin{aligned} x_k(t+1) =& (A + \Delta A(t))x_k(t) + (B + \Delta B(t))u_k(t),\\ y_k(t) =& Cx_k(t),\\ x_k(0) =& x_0, \end{aligned} \tag{1}$$

where $x_k(t) \in \mathbb{R}^{n_x}$ is the state vector, $y_k(t) \in \mathbb{R}^{n_y}$ is the output vector, $u_k(t) \in \mathbb{R}^{n_u}$ is the control input vector. $\{A, B, C\}$ are the nominal batch process matrices with compatible dimensions. The time-varying uncertainties associated with the process dynamics are modeled as additive perturbations and assumed to be of norm-bounded form

$$\Delta A(t) = E\delta(t)F_1,\ \Delta B(t) = E\delta(t)F_2, \tag{2}$$

where E, F_1, and F_2 are known real constant matrices of compatible dimensions, and $\delta(t)$ is an unknown and time-varying perturbation satisfying $\delta^T(t)\delta(t) \preceq I, \forall t \in [0, N-1]$. Regarding the batch process model of (1), the following assumptions are made.

Assumption 1: After each batch, the process resets to the same initial value x_0, and there is no loss of generality in assuming that $x_0 = 0$.

Assumption 2: The matrix pair (A, B) is assumed to be controllable and $\det(CB) \neq 0$.

Remark 1: It is assumed that the matrix C in (1) contains no parameter uncertainty. Otherwise, there will be coupling between uncertain matrices, and hence, LMI-based formulations are difficult to obtain.

Given the desired trajectory y_d, the tracking error on batch k, i.e., the difference between the desired and actual process outputs, is

$$e_k(t) = y_d(t) - y_k(t),\ t \in [0, N-1].$$

The objective of this paper is to use the tracking error to construct a control sequence $\{u_k\}_{k\geq 1}$ such that the uncertain process output y_k tracks the desired trajectory y_d as precisely as possible as the batch index $k \to \infty$. In this case, the tracking error in k converges to zero (or within some specified tolerance), and the tracking performance in N is improved. These requirements are represented mathematically as

$$\begin{aligned} &\lim_{k\to\infty} \|e_k(\cdot)\| = 0,\\ &\lim_{k\to\infty} \|u_k(\cdot) - u_\infty(\cdot)\| = 0, \end{aligned}$$

where $\|\cdot\|$ denotes the norm on the underlying function space and $u_\infty(\cdot)$ is termed the learned control. To achieve the robust convergence of the uncertain linear batch process (1), an ILC law is used to compute the subsequent trial input as the sum of the previous trial input plus a correction. The ILC law, therefore, has the form

$$u_{k+1}(t) = u_k(t) + \Delta u_k(t), \tag{3}$$

where the update $\Delta u_k(t)$ is calculated using the previous batch data.

Analysis in this paper uses incremental variables, i.e.,

$$\begin{aligned} \overline{x}_{k+1}(t) &= x_{k+1}(t) - x_k(t),\\ \overline{u}_{k+1}(t) &= u_{k+1}(t) - u_k(t), \end{aligned} \tag{4}$$

giving the following description of the uncontrolled dynamics

$$\begin{aligned} \overline{x}_k(t+1) =& (A + \Delta A(t))\overline{x}_k(t) + (B + \Delta B(t))\overline{u}_k(t),\\ e_{k+1}(t) =& C\overline{x}_k(t) + e_k(t). \end{aligned} \tag{5}$$

The control law is of the PD-type, i.e., the update term in ILC law (3) takes the form

$$\begin{aligned} \Delta u_k(t) =& K_1\overline{x}_{k+1}(t) + K_2 e_k(t+1)\\ &- K_3(e_{k+1}(t) - e_k(t)), \end{aligned} \tag{6}$$

where K_1, K_2, and K_3 are gain matrices of compatible dimensions to be designed.

Remark 2: The particular choice of the matrices in (6) corresponds to different forms of ILC laws. In particular, the following control laws result.

1) PD-type when $K_2 \neq K_3$,
2) D-type when $K_2 = 0$ and $K_2 \neq K_3$,
3) P-type when $K_2 = K_3$.

III. Embedding ILC into a 2D Systems Setting

This paper uses 2D systems theory to design a control law of the form given in the previous section. The two directions of information propagation are batch to batch (trial to trial) and within a batch. To write the dynamics as a 2D system substitute (6) into (5) and let $\overline{K}_2 = K_2 - K_3$. Then, the controlled dynamics are represented by the following model

$$\begin{aligned} \begin{bmatrix} \overline{x}_k(t+1) \\ e_k(t) \end{bmatrix} =& \mathcal{A}_{11} \begin{bmatrix} \overline{x}_k(t) \\ e_k(t-1) \end{bmatrix} + \mathcal{A}_{12} e_k(t),\\ e_{k+1}(t) =& \mathcal{A}_{21} \begin{bmatrix} \overline{x}_k(t) \\ e_k(t-1) \end{bmatrix} + \mathcal{A}_{22} e_k(t), \end{aligned} \tag{7}$$

where

$$\begin{aligned} \mathcal{A}_{11} =& (A_{11} + \Delta A_{11}),\ \mathcal{A}_{12} = (A_{12} + \Delta A_{12}),\\ \mathcal{A}_{21} =& (A_{21} + \Delta A_{21}),\ \mathcal{A}_{22} = (A_{22} + \Delta A_{22}) \end{aligned} \tag{8}$$

and

$$\mathcal{A}_{11}=\begin{bmatrix} A+BK_1 & BK_3 \\ 0 & 0 \end{bmatrix},\ \mathcal{A}_{12}=\begin{bmatrix} B\overline{K}_2 \\ I \end{bmatrix},$$
$$\mathcal{A}_{21}=\begin{bmatrix} -C(A+BK_1) & -CBK_3 \end{bmatrix}, \mathcal{A}_{22}=I-CB\overline{K}_2,$$
$$\Delta\mathcal{A}_{11}=\begin{bmatrix} E \\ 0 \end{bmatrix}\delta(t)\left(\begin{bmatrix} F_1 & 0 \end{bmatrix}+F_2\begin{bmatrix} K_1 & K_3 \end{bmatrix}\right),$$
$$\Delta\mathcal{A}_{12}=\begin{bmatrix} E \\ 0 \end{bmatrix}\delta(t)F_2\overline{K}_2,\ \Delta\mathcal{A}_{22}=-CE\delta(t)F_2\overline{K}_2,$$
$$\Delta\mathcal{A}_{21}=-CE\delta(t)\left(\begin{bmatrix} F_1 & 0 \end{bmatrix}+F_2\begin{bmatrix} K_1 & K_3 \end{bmatrix}\right).$$

The state-space model (7) is a particular case of a linear 2D Roesser [15] model with parameter uncertainty. Hence, it is termed the equivalent 2D model of the uncertain discrete-time batch processes controlled by application of (6). The matrices $\mathcal{A}_{12}$ and $\mathcal{A}_{21}$ in (7) describe the contributions of the previous batch error to the current batch state and error, respectively. This interaction is the source of the unique batch process control problem and can result in oscillations that increase in amplitude from batch to batch, i.e., with increasing k.

Robust stability of the controlled dynamics is enforced by (structural) stability of the 2D model (7). This property will result in batch-to-batch error convergence for all admissible uncertainties. Also, the design of ILC laws can be completed with the 2D system formulation, as shown in the remainder of this section and the next.

To simplify the presentation, the following notation is introduced

$$\begin{gathered}\overline{K}_1=\begin{bmatrix} K_1 & K_3 \end{bmatrix}, \overline{C}_1=\begin{bmatrix} CA & 0 \end{bmatrix}, \overline{F}_1=\begin{bmatrix} F_1 & 0 \end{bmatrix},\\ \overline{E}=\begin{bmatrix} E \\ 0 \end{bmatrix}, \overline{B}_1=\begin{bmatrix} 0 \\ I \end{bmatrix}, \overline{B}=\begin{bmatrix} B \\ 0 \end{bmatrix},\ \overline{A}=\begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}.\end{gathered} \tag{9}$$

Hence

$$\begin{aligned}\mathcal{A}_{11}&=\overline{A}+\overline{B}\overline{K}_1, \mathcal{A}_{21}=-\overline{C}_1-CB\overline{K}_1,\\ \mathcal{A}_{12}&=\overline{B}_1+\overline{B}\overline{K}_2, \mathcal{A}_{22}=I-CB\overline{K}_2,\\ \Delta\mathcal{A}_{11}&=\overline{E}\delta(t)\left(\overline{F}_1+F_2\overline{K}_1\right), \Delta\mathcal{A}_{22}=-CE\delta(t)F_2\overline{K}_2,\\ \Delta\mathcal{A}_{21}&=-CE\delta(t)\left(\overline{F}_1+F_2\overline{K}_1\right), \Delta\mathcal{A}_{12}=\overline{E}\delta(t)F_2\overline{K}_2.\end{aligned} \tag{10}$$

A. Robust structural stability of a linear 2D model

Robust structural stability of the Roesser model (7) is characterized by the following lemma.

Lemma 2: (see [9], [10] and references therein) The 2D Roesser model of the controlled ILC dynamics given in (7) is robustly structurally stable if and only if the following conditions hold

i) $\forall\lambda\in\overline{\mathbb{D}}$, $\det(\lambda I-\mathcal{A}_{22})\neq 0$,

ii) $\forall\lambda\in\partial\overline{\mathbb{D}}$, $\det(G(\lambda))\neq 0$,

for all admissible uncertainties and where $\overline{\mathbb{D}}=\{z\in\overline{\mathbb{C}}, |z|\geq 1\}$, $\partial\overline{\mathbb{D}}=\{z\in\overline{\mathbb{C}}, |z|=1\}$ and

$$G(\lambda)=\mathcal{A}_{21}(\lambda I-\mathcal{A}_{11})\mathcal{A}_{12}+\mathcal{A}_{22}.$$

Condition $i)$ in this last result is relatively easily transformed into a condition for the robust stability of a standard (sometimes termed 1D in the 2D systems literature) system that LMI conditions can efficiently check. The main difficulty is the computational cost associated with the condition $ii)$. This condition requires computations for all $\forall\lambda\in\partial\overline{\mathbb{D}}$ and clearly, the number of computations increases without bound, so the LMI-based formulation of condition $ii)$ cannot be easily provided. However, using results in [9] it follows immediately that the conditions $i)$ and $ii)$ in Lemma 2 can be replaced by the following inequalities

$$\begin{bmatrix} \mathcal{A}_{22} \\ I \end{bmatrix}^T (R\otimes P)\begin{bmatrix} \mathcal{A}_{22} \\ I \end{bmatrix}\prec 0 \tag{11}$$

and

$$\begin{bmatrix} G(\lambda) \\ I \end{bmatrix}^* (R\otimes P(\lambda))\begin{bmatrix} G(\lambda) \\ I \end{bmatrix}\prec 0, \tag{12}$$

where $R=\operatorname{diag}\{1,-1\}$ and the matrices P and $P(\lambda)$ satisfy $P\succ 0$ and $P(\lambda)\succ 0\ \forall\lambda\in\partial\overline{\mathbb{D}}$.

The inequality (11) implies that $\rho(\mathcal{A}_{22})\leq 1$ for all admissible uncertainties and (12) can be transformed into $\rho(G(\lambda))\leq 1\ \forall\lambda\in\partial\overline{\mathbb{D}}$. Moreover, as $G(\lambda)$ and $P(\lambda)$ depend on λ, then a sequence of transformations that leads to LMI formulation of (12) is required. To proceed, note that the existence of a matrix $P(\lambda)\succ 0$ implies that there exists a matrix $Q(\lambda)$ satisfying $P(\lambda)=\operatorname{sym}\{\mathrm{Q}(\lambda)\}$. Consequently, the inequality (12) can be written as

$$G(\lambda)^*(Q(\lambda)+Q^*(\lambda))G(\lambda)-(Q(\lambda)+Q^*(\lambda))\prec 0$$

or

$$\begin{bmatrix} M(\lambda) \\ I \end{bmatrix}^*\begin{bmatrix} 0 & I & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & -I \\ 0 & 0 & -I & 0 \end{bmatrix}\begin{bmatrix} M(\lambda) \\ I \end{bmatrix}\prec 0, \tag{13}$$

where $M(\lambda)=[G^*(\lambda)Q(\lambda)\ G^*(\lambda)\ Q^*(\lambda)]^*$.

The remaining problem is the dependence of $Q(\lambda)$ on the parameter λ. This complex dependence prevents finding the feasible solution to inequalities (12) and (13). Using the results of [16], however, the following theorem can be established.

Theorem 1: Assume that (12) has feasible solution for some $P(\lambda)$. This there exists $\alpha\in\left[0; b=\frac{n_y}{2}((n_x+n_y)^2+(n_x+n_y)-2)\right]$ such that $P(\lambda)$ can be taken to have the form

$$P(\lambda)=\operatorname{sym}\left\{\sum_{h=0}^{\alpha}Q_h\lambda^h\right\}=\Upsilon^*(\lambda)\mathcal{Q}\Upsilon(\lambda),$$

with

$$\mathcal{Q}=\left[\begin{array}{c|c} \operatorname{sym}\{Q_0\} & Q_1\dots Q_\alpha \\ \hline Q_1^* & \\ \vdots & 0 \\ Q_\alpha^* & \end{array}\right], \Upsilon(\lambda)=\begin{bmatrix} \lambda^0 I_{n_x+n_y} \\ \lambda^1 I_{n_x+n_y} \\ \vdots \\ \lambda^\alpha I_{n_x+n_y} \end{bmatrix} \tag{14}$$

and $Q_h\in\mathbb{R}^{(n_x+n_y)\times(n_x+n_y)}$, $\quad h=0,\dots,\alpha$.

Proof: The proof of this theorem follows that of Theorem 2 in [16]. ∎

The task now is to convert the inequality (13) into the LMI-based condition and compute ILC law matrices. Firstly, it is evident that $Q(\lambda)$ can be rewritten as

$$Q(\lambda) = \lambda I \star \left[\begin{array}{c|c} A_\Upsilon & B_\Upsilon \\ \hline C_\Upsilon & D_\Upsilon \end{array}\right],$$

where the matrices A_Υ, B_Υ, C_Υ and D_Υ depend on the parameter $\alpha \geq 0$. Setting $\alpha = 0$ (the lowest computational burden) yields

$$\left[\begin{array}{c|c} A_\Upsilon & B_\Upsilon \\ \hline C_\Upsilon & D_\Upsilon \end{array}\right] = \left[\begin{array}{c|c} [\,]_{0,0} & [\,]_{0,m} \\ \hline [\,]_{m,0} & I_m \end{array}\right],$$

where $m = n_x + n_y$. Next, letting $\alpha > 0$ (increasing the computational burden), gives

$$A_\Upsilon = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & 0 & \ddots & 1 & 0 \\ \vdots & \vdots & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix} \otimes I_m,\ C_\Upsilon = \begin{bmatrix} 0_{m,\alpha m} \\ \begin{bmatrix} 0 & & 1 \\ & \iddots & \\ 1 & & 0 \end{bmatrix}_{\alpha\times\alpha} \otimes I_m \end{bmatrix},$$
$$B_\Upsilon = \begin{bmatrix} 0_{(\alpha-1)m,m} \\ I_m \end{bmatrix},\ D_\Upsilon = \begin{bmatrix} I_m \\ 0_{\alpha m,m} \end{bmatrix}. \tag{15}$$

Moreover, it can be established that

$$M(\lambda) = \lambda I \star \left[\begin{array}{c|c} A_M & B_M \\ \hline C_M & D_M \end{array}\right],$$

where

$$A_M = \begin{bmatrix} A_\Upsilon & 0 & B_\Upsilon \mathcal{A}_{12} \\ 0 & A_\Upsilon & 0 \\ 0 & 0 & \mathcal{A}_{22} \end{bmatrix}, B_M = \begin{bmatrix} B_\Upsilon \mathcal{A}_{11} \\ B_\Upsilon \\ \mathcal{A}_{22} \end{bmatrix},$$
$$C_M = \begin{bmatrix} C_\Upsilon & 0 & D_\Upsilon \mathcal{A}_{12} \\ 0 & C_\Upsilon & 0 \end{bmatrix}, D_M = \begin{bmatrix} D_\Upsilon \mathcal{A}_{11} \\ D_\Upsilon \end{bmatrix}.$$

The following result permits ILC law design by providing an equivalent formulation to conditions $i)$ and $ii)$ of Lemma 2.

Theorem 2: Suppose that an ILC law (6) is applied to the system (1). Then, the resulting ILC dynamics are described as a 2D Roesser model of the form (7) is robustly structurally stable, and hence batch-to-batch error convergence occurs for all admissible uncertainties if and only if there exists an integer $\alpha \in \left[0; b = \frac{n_y}{2}((n_x+n_y)^2 + (n_x+n_y) - 2)\right]$, Q_h, $h = 0, \dots, \alpha$, $X \succ 0$ and $Y \succ 0$ such that

$$\begin{bmatrix} I & 0 \\ A_M & B_M \\ C_M & D_M \end{bmatrix}^T \begin{bmatrix} \hat{R}\otimes X & 0 \\ 0 & R\otimes \mathcal{Q} \end{bmatrix} \begin{bmatrix} I & 0 \\ A_M & B_M \\ C_M & D_M \end{bmatrix} \prec 0 \tag{16}$$

and

$$\begin{bmatrix} I & 0 \\ A_\Upsilon & B_\Upsilon \\ C_\Upsilon & D_\Upsilon \end{bmatrix}^T \begin{bmatrix} \hat{R}\otimes Y & 0 \\ 0 & -\mathcal{Q} \end{bmatrix} \begin{bmatrix} I & 0 \\ A_\Upsilon & B_\Upsilon \\ C_\Upsilon & D_\Upsilon \end{bmatrix} \prec 0 \tag{17}$$

hold where $R = \mathrm{diag}\{1, -1\}$, $\hat{R} = \mathrm{diag}\{-1, 1\}$.

Proof: The proof is by employing the same steps to prove the result for the 2D linear model in [17], [9]. Hence, the details are omitted. ■

Theorem 2 is a relaxation of condition $ii)$ of Lemma 2 through the S-procedure described in [18]. In what follows, condition $ii)$ of Lemma 2 is implied by condition (16).

Product terms exist between the matrix variables X, Y and the control law matrices K_1, K_2 and K_3 in the conditions of Theorem 2 and hence the ILC law design procedures (the conditions (16) and (17) are not LMIs). In the following, decoupling matrix variables and ILC law matrices enable a more easily tractable condition for ILC law design to be established.

IV. MAIN RESULTS

This section aims to develop a new robust ILC scheme design procedure for uncertain batch processes (1) for which the following notation is used

$$\Lambda = \begin{bmatrix} I_\nu & 0 & 0 & 0 & 0 & 0 \\ 0 & I_\nu & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{n_y} & 0 & 0 & 0 \\ A_\Upsilon & 0 & 0 & 0 & 0 & B_\Upsilon \\ 0 & A_\Upsilon & 0 & B_\Upsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{n_y} & 0 \\ C_\Upsilon & 0 & 0 & 0 & 0 & D_\Upsilon \\ 0 & C_\Upsilon & 0 & D_\Upsilon & 0 & 0 \end{bmatrix}, \mathcal{A} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I_{n_y} & -\overline{C}_1 \\ \overline{B}_1 & \overline{A}_1 \\ -I_{n_y} & 0 \\ 0 & -I_{n_x+n_y} \end{bmatrix},$$
$$\mathcal{B}^T = \begin{bmatrix} 0 & 0 & -(CB)^T & \overline{B}^T & 0 & 0 \end{bmatrix},$$

where ν (which appears as dimensions of some blocks in Λ) must have the same dimensions the matrix A_Υ defined in (15). Furthermore, let us assume that the 2D system of (7) is (structurally) stable, which means that the control law (6) stabilizes the resulting dynamics. Also, let the scalar parameters β_1 and β_2 be freely chosen from the set $\mathbb{D} = \{z \in \mathbb{C}, |z| < 1\}$ (i.e., inside of the open unit disc). This choice immediately implies that the 2D system model

$$\left[\begin{array}{c} \overline{x}_k(t+1) \\ e_k(t) \\ \hline e_{k+1}(t) \end{array}\right] = \left[\begin{array}{c|c} \beta_1 I_{n_x+n_y} & 0 \\ \hline 0 & \beta_2 I_{n_y} \end{array}\right] \left[\begin{array}{c} \overline{x}_k(t) \\ e_k(t-1) \\ \hline e_k(t) \end{array}\right] \tag{18}$$

is also (structurally) stable. Clearly, these 2D systems must satisfy Theorem 2 and, according to the analysis in [16], they must also share the same matrix $P \succ 0$. Hence, it remains to focus on (16) only - the condition (17) is not required, but it may introduce a slight level of conservatism (and hence the conditions will be the sufficient ones only). Moreover, for known choice of scalars β_1 and β_2 (e.g. $\beta_1 = \beta_2 = 0$) the following matrix can be defined

$$L_\beta = \begin{bmatrix} 0 & 0 & \beta_2 I_{n_y} & 0 & -I_{n_y} & 0 \\ 0 & 0 & 0 & \beta_1 I_{n_x+n_y} & 0 & -I_{n_x+n_y} \end{bmatrix}.$$

The following theorem is the significant new result of this paper's ILC design, based on the developed conditions for robust stabilization of the equivalent 2D model.

Theorem 3: Consider an uncertain batch process described by the version of (1) with uncertainty described by (2). Furthermore, suppose that an ILC law (6) is applied to the system (1). Then the resulting ILC scheme described as a 2D Roesser model of the form (7) is structurally robustly stable, and hence batch-to-batch error convergence occurs for all admissible uncertainties, if an integer $\alpha \in \left[0; b = \frac{n_y}{2}((n_x+n_y)^2 + (n_x+n_y)-2)\right]$ can be found such that there exist matrices Q_h, $h = 0, \ldots, \alpha$, $P \succ 0$, M, S and a positive scalar ϵ such that the following LMI

$$\begin{bmatrix} \Omega & \epsilon\mathcal{E} & \mathcal{H}^T \\ \epsilon\mathcal{E}^T & -\epsilon I & 0 \\ \mathcal{H} & 0 & -\epsilon I \end{bmatrix} \prec 0 \tag{19}$$

is feasible and where the matrices R and $\hat{R}$ are as in (16)-(17) and

$$\Omega = \Lambda^T \begin{bmatrix} \hat{R} \otimes P & 0 \\ 0 & R \otimes \mathcal{Q} \end{bmatrix} \Lambda + \text{sym}\left\{(\mathcal{A}M + \mathcal{B}S)L_\beta\right\},$$

$$\mathcal{E}^T = \begin{bmatrix} 0 \; 0 \; -(CE)^T \; \overline{E}^T \; 0 \; 0 \end{bmatrix},$$

$$\mathcal{H} = \left(F_2 S + \overline{F}_1 \begin{bmatrix} 0_{n_x+n_y, n_y} & I_{n_x+n_y} \end{bmatrix} M\right) L_\beta.$$

Moreover, if the LMI (19) is feasible, the required control law matrices K_1, K_2 and K_3 are computed as

$$\begin{bmatrix} \overline{K}_2 \,|\, K_1 \; K_3 \end{bmatrix} = SM^{-1}, \; K_2 = \overline{K}_2 + K_3. \tag{20}$$

Proof: Suppose that the LMI (19) is feasible. Then, application of Schur's complement formula to (19) gives

$$\Omega + \epsilon\mathcal{E}\mathcal{E}^T + \epsilon^{-1}\mathcal{H}^T\mathcal{H} \prec 0.$$

Next, assign $\mathcal{Z} \leftarrow \Omega$, $\mathcal{X} \leftarrow \mathcal{E}$, $\mathcal{Y} \leftarrow \mathcal{H}$ and by Lemma 1 the last inequality is feasible if and only if

$$\Omega + \text{sym}\left\{\mathcal{E}\delta(t)\mathcal{H}\right\} \prec 0.$$

The remaining part of the proof follows immediately from the proof of Theorem 5 in [9] and hence the details are omitted. ■

Remark 3: Since ILC law matrices K_1, K_2 and K_3 must be real, the matrix variables M and S must also be real, which could theoretically induce a slight conservatism (Note that the less conservative LMI-based results on stability 2D system are originally formulated with Hermitian matrices).

V. Numerical Case Study

The example considers the linearized dynamics of the injection molding process given in [6], [19]. This application is a typical batch process where nozzle pressure is a critical process variable to be controlled. When considering the nozzle pressure response to the hydraulic control valve opening, the following model was identified

$$y_k(t) = \frac{1.239(\pm 5\%)z^{-1} - 0.9282(\pm 5\%)z^{-2}}{1 - 1.607(\pm 5\%)z^{-1} + 0.6086(\pm 5\%)z^{-2}} u_k(t),$$

where the percentages in parentheses indicate the parameter perturbations in the worst case of cyclic operation. This model can be rewritten in the form of (1) as

$$\begin{aligned} x_k(t+1) &= \left(\begin{bmatrix} 1.607 & 1 \\ -0.6086 & 0 \end{bmatrix} + \Delta A(t)\right) x_k(t) \\ &+ \left(\begin{bmatrix} 1.239 \\ -0.9282 \end{bmatrix} + \Delta B(t)\right) u_k(t), \\ y_k(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_k(t), \end{aligned}$$

where

$$\Delta A(t) = \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}^{E} \begin{bmatrix} \delta(t) & 0 \\ 0 & \delta(t) \end{bmatrix} \overbrace{\begin{bmatrix} 0.0804 & 0 \\ -0.0304 & 0 \end{bmatrix}}^{F_1}$$

$$\Delta B(t) = \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}^{E} \begin{bmatrix} \delta(t) & 0 \\ 0 & \delta(t) \end{bmatrix} \overbrace{\begin{bmatrix} 0.062 \\ -0.0462 \end{bmatrix}}^{F_2}$$

In this case $n_x = 2$ and $n_y = 1$, and hence $\alpha \in [0, 5]$. The chosen desired trajectory is

$$y_d(t) = \begin{cases} 200, & 1 \leq t < 100; \\ 200 + 5(t - 100), & 100 \leq t < 120; \\ 300, & 120 \leq t \leq N = 200. \end{cases}$$

For practical implementation, the initial part of $y_d(t)$ is pre-filtered by $G_f = (z^{-1} + z^{-2})/(3 - z^{-1})$, where it is essential to note that this step only applies to this application. Furthermore, the Root Mean Square Error (RMSE) value of the tracking error defined by

$$\text{RMSE}(e_k) = \sqrt{\frac{1}{201}\sum_{t=0}^{200} e_k^2(t)}$$

plotted against k is adopted as a performance index to evaluate the tracking performance. Also, it should be emphasized that there are many articles with control results for this batch process. Still, none focused exclusively on uncertainties (often, the design procedures were supported by optimizing selected control indices, e.g., the H_∞ norm), so it is not easy to find examples for direct comparison. Here, a comparison with the results in [19] is undertaken.

Applying Theorem 3 for $\beta_1 = 0.8$, $\beta_2 = 0$ and $\alpha = 4$ gives ILC law matrices in (6) as

$$K_1 = [-1.3068 \;\; -0.8133],\; K_2 = 0.7931,\; K_3 = -0.0018.$$

It can be seen from Figure 1 that batch-to-batch error convergence occurs. The effectiveness of the new ILC design is apparent since the RMSE of the new design is lower when compared with [19]. Additionally, Figure 2 shows

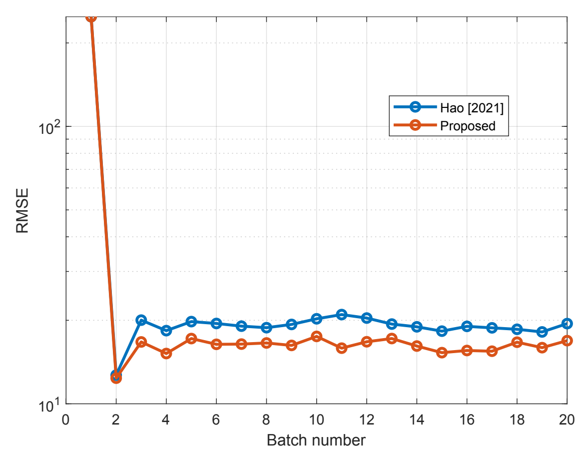

Fig. 1: RMSE(e_k) values for the different designs.

the corresponding spectral radius of the transfer function $G(\lambda)\ \forall\lambda \in \partial\overline{\mathbb{D}}$. From this comparison of the two ILC law designs in Figure 2, it is evident that the new design can produce an ILC law with the lowest level for spectral radius for extreme (min and max) values of the uncertainty and hence can deliver faster batch-to-batch error convergence. Consider the nominal case (no uncertainty). Then Figure 3

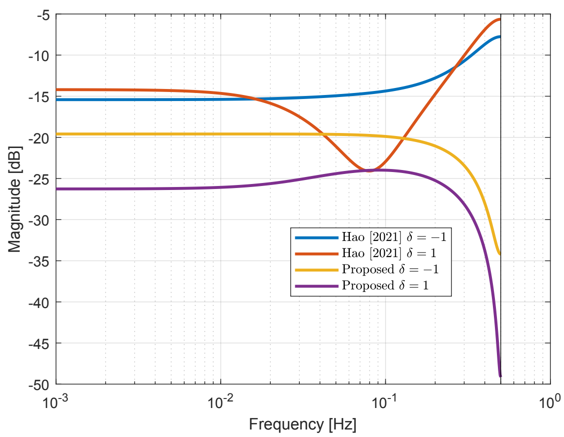

Fig. 2: Plot of the spectral radius of $G(\lambda)$ (uncertain process case).

shows the corresponding spectral radius. Although for very low frequencies, the result of [19] is better than the one developed in this paper, the new design generates solutions that maintain low gain (up to the error from the previous batch) for the entire frequency spectrum, and therefore achieve faster convergence.

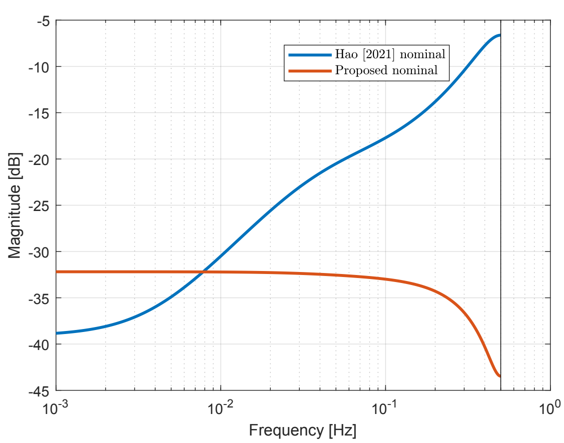

Fig. 3: Plot of the spectral radius of $G(\lambda)$ (nominal process case)

VI. Conclusions and Future Research

In this paper, new results on the ILC law design problem for a class of uncertain batch processes have been developed. These results have been obtained by transforming the initial problem into an equivalent one of designing robustly stabilizing state feedback gains for 2D linear systems described by the Roesser model. Sufficient conditions for the existence of a robustly convergent ILC law have been obtained in terms of the corresponding LMIs. A simulation study based on the nozzle velocity control system of an injection molding process verifies the effectiveness of this design method. Future work aims to extend results to continuous-time batch processes and more complex control laws that use only measured outputs.

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